The Hawking Radiation as Gravitational Fowler-Nordheim Emission in an Uniformly Accelerated Frame, in the Non-Relativistic Scenario

A part of Ph.D. work of Sanchari De

- Black Holes:

1. A black hole is defined as a region of spacetime that can not communicate with the external universe.

2. Stellar Black Holes: End product of massive stars $M > 20M_{\odot}$.

3. Primordial black hole: Formed in the early universe due to some cosmological events (density fluctuation).

4. Massive black holes: Present at the centre of galaxies- the reason of their formation is not exactly known.

5. Mini or micro black holes: Formed in ultra-relativistic heavy-ion collisions and also have primordial origin.

- Hawking Radiation:

  - To describe a black hole one needs only a few parameters, such as mass, angular momentum and possibly charge. $\Longrightarrow$ Gravitational collapse washes out initial conditions -the detailed properties of the initial configurations, such as geometrical shape, composition of the constituent matter, etc. are obliterated.

  - Gravitational collapse is accompanied by a loss of information about the system. This situation suggests that one might be able to associate the concept of temperature with a black hole.

  - Further hints at the possible utility of thermodynamic concepts is provided by the law of non-decrease of the black hole surface area (the famous area theorem of Hawking, C. Bekenstein, PR D12, 3077 (1975)), which bears some analogy to the non-decrease of entropy- black-hole thermodynamics.

- At the level of purely classical physics, this analogies do not lead to any fruitful consequences. If a temperature is associated with the black hole, then it must be able to emit black body radiation. But according to classical physics no radiation can escape from a black hole. The concept of black hole temperature has no use in classical physics.

- Situation is quite different in quantum theory. Quantized radiation can escape from a black hole, despite the existence of potential barrier $\Longrightarrow$ Quantum Black Holes.

- The quantum mechanical tunneling effect is responsible for black hole emissions. Hence Hawking introduced a temperature associated with the evaporating black hole, known as Hawking temperature ($T = \hbar/(8\pi k M$). (S.W. Hawking, Nature 248, 30 (1974); Comm. Math. Phys. 43, 199 (1975); PR D14, 2460 (1976)).

- During the emission of thermal quanta from black holes (evaporation), M decreases by energy conservation and thus so does A (and S) $\Longrightarrow$ violates Hawking's area theorem.

- The area theorem of classical general relativity gets replaced by a generalized second law of thermodynamics: in any interactions, the sum of entropies of all black holes plus the entropy of matter outside black holes never decrease. Thus black holes actually fit very naturally into an extended framework of thermodynamics.

- Black hole evaporation can be understood as pair creation in the gravitational field of the black hole, one member of the pair going down the black hole and the other coming out to infinity.

- In quantum field theory, the vacuum is continuously undergoing fluctuations, where a pair of virtual particles is created and then annihilates. If the field is strong enough, the particles tunnel through the quantum barrier and materialize as real particles.

- The critical field strength is achieved when the work done in separating them by the Compton wavelength equals the energy necessary to create the particles.

- Event Horizon: A black hole is defined as a region of spacetime that can not communicate with the external universe. The boundary region is called the surface of the black hole, or the event horizon.

- Schwinger's Mechanism: The Schwinger mechanism refers to the production of charged fermion—anti-fermion pairs out of the vacuum by a static external electric field. This is essentially the dielectric breakdown of the vacuum.

- The quantum vacuum is unstable under the influence of an external electric field, as the virtual electron-positron pairs can gain energy from the external field. If the field is sufficiently strong, these virtual particles can gain the threshold pair creation energy $2mc^2$ and become real electron-positron pairs. This remarkable phenomenon was first predicted by Heisenberg and his student Hans Euler in 1936, based on work of Sauter in 1931, and later formalized in the language of QED by Schwinger in 1951. However, the electric field required to see this effect is astronomically huge, $E_{\text{critical}} \sim 10^{16}$V/cm, and so it has not yet been directly observed, even using the strongest lasers.

- Theoretically, this is a non-perturbative effect, as the virtual particles tunnel out of the Dirac sea. This makes this elusive effect of great interest for other theories, such as quantum chromodynamics (QCD), where non-perturbative effects are known to be significant but are not directly accessible, and also in gravitational physics, in particular for the phenomena of Hawking radiation near a black hole, and Unruh radiation of accelerating particles / frame of references.

- Because of identical nature of Schwinger mechanism of pair production in presence of strong electric field and the Hawking radiation at the event horizon of a black hole, in the conventional scenario, the Hawking radiation is therefore believed to be a tunneling process at the event horizon.

- The strong electric field which separates two opposite charged particles beyond their Compton wavelength in the Schwinger process is replaced by the event horizon in the case of Hawking radiation.

- Unruh Effect: Further, the Hawking radiation may be interpreted as the outcome of Unruh effect (Unruh W.G., 1976, Phys. Rev. D14, 4i; 1976, Phys. Rev. D14, 870).

- The argument of Unruh for such emission process is that an observer in an accelerated frame will see radiation in the vacuum of inertial observer (known as Unruh effect). Whereas from inertial frame, there will be no radiation in the vacuum states.

- The Unruh effect predicts that an accelerating observer will see black-body radiation in a true vacuum of an inertial observer. The temperature of the inertial vacuum as measured by the accelerated observer increases with the magnitude of acceleration and is given by $T = T_U = \hbar\alpha/(2\pi ck)$, the Unruh temperature.

- In other words, the background appears to be warm from an accelerating reference frame. The ground state for an inertial observer is seen as in thermodynamic equilibrium with a non-zero temperature by the accelerated observer.

- Principle of Equivalence: An accelerated frame of reference is equivalent to a frame at rest in presence of gavitational field.

- Near the event horizon of a blackhole the gravitational field is extremely high is equivalent to a frame undergoing strong accelerated motion.

- Although in quantum field theoretic approach the Schwinger mechanism is generally used to explain the tunneling process. However, in a reference frame undergoing a uniform accelerated motion in an otherwise flat Minkowski space-time geometry, in the non-relativistic approximation, the Hawking radiation is very much analogous to Fowler-Nordheim field emission, the typical electron emission process from a metal surface under the action of external electrostatic field, even at extremely low temperature.

- In the non-relativistic approximation the concept of quantum vacuum does not exist. The formalism has therefore been developed with the starting point immediately after the emission.

- The Fowler-Nordheim emission is an electron emission process induced by a strong external electrostatic field applied at the metal surface, acts as a driving force to liberate electrons from metal surface. (Fowler R.H. and Nordheim Dr. L., 1928, Proc. R. Soc. London 119, 173)

- This Emission can only be explained as the quantum mechanical tunneling of electrons through surface barrier.

Relativistic Quantum Field Theoretic Approach of Black Hole Emission in Curved Space Time: Schwinger's Mechanism

In the Non-Relativistic Approximation, in a Reference Frame Undergoing a Uniform Accelerated Motion in an Otherwise Flat Minkowski Space-Time Geometry, the Hawking Radiation is Very Much Analogous to Fowler-Nordheim Emission (Fowler-Nordheim / Field / Cold Emission).

Rindler Coordinate

The Rindler space-time coordinates are the just an uniformly accelerated frame transformation of the Minkowski metric of special relativity.

$$ct = \left(\frac{c^2}{g} + x'\right) \sinh\left(\frac{gt'}{c}\right) \quad \text{and} \quad x = \left(\frac{c^2}{g} + x'\right) \cosh\left(\frac{gt'}{c}\right)$$

Hence one can also express the inverse relations

$$ct' = \frac{c^2}{2g} \ln\left(\frac{x + ct}{x - ct}\right) \quad \text{and} \quad x' = (x^2 - (ct)^2)^{1/2} - \frac{c^2}{g}$$

(Birrell N.D. and Davies P.C.W., 1982, Quantum Field Theory in Curved Space, Cambridge University Press, Cambridge; Rindler, W. 1977, Essential Relativity, Springer-Verlag, New York; Weinberg, S. 1972, Gravitation and Cosmology, Wiley, New York; Misner, C.W., Thorne, Kip S. and Wheeler, J.A. 1972, Gravitation, W.H. freeman and Company, New York.)

The motion is assumed to be rectilinear and along x-direction, $dy' = dy$ and $dz' = dz$. the Rindler coordinate transform the Minkowski line element

$ds^2 = d(ct)^2 - dx^2 - dy^2 - dz^2$ to $ds^2 = \left(1 + \frac{gx'}{c^2}\right)^2 d(ct')^2 - dx'^2 - dy'^2 - dz'^2$

To develop the quantum mechanical formalism for a particle undergoing an uniform accelerated motion, we start with the single particle classical Lagrangian:

The action integral may be written as: (Landau L.D. and Lifshitz E.M., 1975, The Classical Theory of Fields, Butterworth-Heimenann, Oxford).

$$S = -\alpha_0 \int_a^b ds \equiv \int_a^b L dt$$

Hence for $\alpha_0 = m_0 c^2$

$$L = -m_0 c^2 \left[\left(1 + \frac{\alpha x}{c^2}\right)^2 - \frac{v^2}{c^2} \right]$$

where $\alpha = g$ is the uniform acceleration of the particle along x-direction, $v = u_x$, the particle velocity and m_0 is the rest mass of the particle.

The three momentum vector of the particle can then be written as

$$\vec{p} = \frac{m_0 \vec{v}}{\left[\left(1 + \frac{\alpha x}{c^2}\right)^2 - \frac{v^2}{c^2}\right]^{1/2}}$$

Hence the Hamiltonian of the particle is given by

$$H = m_0 c^2 \left(1 + \frac{\alpha x}{c^2}\right) \left(1 + \frac{p^2}{m_0^2 c^2}\right)^{1/2}$$

Now in the quantum mechanical picture, the classical dynamical variables x, $\vec{p}$ and H are treated as operators, with the commutation relations

$$[x, p_x] = i\hbar \text{ and } [x, p_y] = [x, p_z] = 0$$

The Schrödinger equation for the particle is then given by

$$H\psi = \left(1 + \frac{\alpha x}{c^2}\right)\left(m_0 c^2 + \frac{p^2}{2m_0}\right)\psi = E\psi$$

Hence

$$-\frac{\hbar^2}{2m_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi(x,y,z) + \frac{\alpha E x}{c^2}\psi = E_k\psi$$

where $E_k = E - m_0 c^2$, the kinetic energy of the particle. It is quite obvious that in the separable form, the solution of the above equation may be written as

$$\psi(x,y,z) = NX(x)\exp\left(-\frac{ip_y y}{\hbar}\right)\exp\left(-\frac{ip_z z}{\hbar}\right)$$

Then we have

$$\frac{d^2X}{dx^2} - \frac{2m_0 E\alpha}{\hbar^2 c^2} x X(x) = -\frac{2m_0}{\hbar^2}\left(E_k - \frac{p_\perp^2}{2m_0}\right)X(x)$$

where

$$\frac{p_\perp^2}{2m_0} = \frac{p_x^2 + p_y^2}{2m_0}$$

is the orthogonal part of kinetic energy.

Hence the parallel part of kinetic energy is given by

$$E_{||} = E_k - \frac{p_\perp^2}{2m_0}$$

Let us put

$$\zeta = \left(\frac{2m_0 E\alpha}{\hbar^2 c^2}\right)^{1/3} x$$

a new dimensionless variable and

$$E' = \frac{2m_0 E_{||}}{\hbar^2}\left(\frac{\hbar^2 c^2}{2m_0 E\alpha}\right)^{2/3}$$

Then it can very easily be shown that the above differential equation reduces to

$$\frac{d^2X}{d\xi^2} + \xi X = 0$$

with $\xi = E' - \zeta$.

This equation is of the same form as was obtained by Fowler and Nordheim in their original work on field emission of electrons

The mathematical reason behind the similar form of the differential equations is because of the same constant type driving fields for both the cases. In the case of Fowler-Nordheim emission, it is the constant attractive electrostatic field obtained from the potential of the form $C - Ex$, where C is the surface barrier, which is approximated with the work function of the metal and E is the uniform electrostatic field near the metal surface. The quantity $C - Ex$ acts as the driving potential for cold emission. Whereas in the case of black hole emission the driving force is the uniform gravitational field near the event horizon of the black hole.

The complete form of solution is given by

$$\psi(x,y,z) = N\,\text{exp}(-ip_y y/\hbar)\,\text{exp}(-ip_z z/\hbar)(E' - \zeta)^{1/2} H^{(2)}_{1/3}\left[\frac{2}{3}(E' - \zeta)^{3/2}\right]$$

The above oscillatory solution along x-direction implies that $E' - \zeta$ must be > 0,

$\Longrightarrow$

$$x < \frac{E_{||}}{E}\frac{c^2}{\alpha}$$

To get some more physical insight from this analysis, we assume that $E \approx E_{||}$. Then we have

$$x < \frac{c^2}{\alpha}$$

If it is further assumed that

$$\alpha \approx \frac{GM}{R^2}$$

We Have

$$x < 2\frac{R^2}{R_s}$$

- where $R_s = 2GM/c^2$, the Schwarzschild radius, M is the mass of the black hole. If we put $R \approx R_s$, then $x < 2R_s$. Therefore in the approximation x lies within the region R_s to $2R_s$ outside the event horizon.

- We may approximate that the emission occurs within the region R_s to $2R_s$.

- In the case of cold emission, the driving force is the strong external electrostatic field applied near the metal surface. These electrons are liberated to the real world from the conduction band of the metal. Further in the case of cold emission, only electrons are liberated.

- Whereas for the black hole emission, it is the strong gravitational field near the event horizon is the driving force. Because of the strong gravitational field the pair of particles and anti-particles tunnel to the real world.

- Unlike the Fowler-Nordheim case, here the emission occurs in pairs and not necessarily electron-positron pairs.

- Further in the case of black hole emission the pairs come out from the quantum vacuum, where they are in the form of condensed phase, whereas electrons in the conduction band are the constituents of degenerate Fermi gas.

- In presence of strong black hole gravitational field near the event horizon, which is equivalent to uniformly accelerated frame without gravity, the temperature of the vacuum will be large enough to create particle and anti-particle pairs if $kT_U > 2m_0c^2$.

- **Final Remark:**

  1. *It has been theoretically established that in the quantum field theoretic approach in curved space-time, the creation of particles at the event horizon, which is basically a quantum tunneling process is identified as Schwinger process.*

  2. *In this article we have observed that in the non-relativistic approximation, it is also a tunneling process, but may be identified as gravitational Fowler-Nordheim emission. However, in the non-relativistic quantum mechanics the concept of vacuum does not exist. Therefore we had to start with the created particle (anti-particle) at the event horizon.*

  3. For black holes of mass $1 \times M_\odot$ or greater, $T \leq 10^{-15}$K $\approx 0 \Longrightarrow$ No Hawking radiation.

  4. For 1K increase in Unruh temperature, the accleration should be $\geq 10^{22}$cm/sec^2.

  5. Possibility of mini black holes in ultra-relativistic heavy ion collisions $\Longrightarrow$ Strongly correlated ideal fluid (quark-gluon plasma)