A Brief Discussion On the Evolution of Quantum Physics

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- Prior to 1900, experimental observations $\longrightarrow$ explained by Newton's laws of motion $\longrightarrow$ both in celestial (Planetary motions- Kepler's laws etc.) and terrestrial (motion under gravity etc.) scales. Light: wave nature from Young's experiment (1803), Maxwell's formalism for light as electromagnetic waves (1804) $\longrightarrow$ **Classical Mechanics**.

- Development of **Special Theory of Relativity** by Albert Einstein (1904) $\longrightarrow$ for the particle with velocity $\sim$ velocity of light

- Geometrical theory of gravity was also developed by Albert Einstein $\longrightarrow$ **General Theory of Relativity**

- These are all well established theories- so far the classical motions of macroscopic (celestial and terrestrial) objects are concerned.

- Difficulty arises with the understanding of experimental data associated with the motions of systems of atomic and sub-atomic dimensions

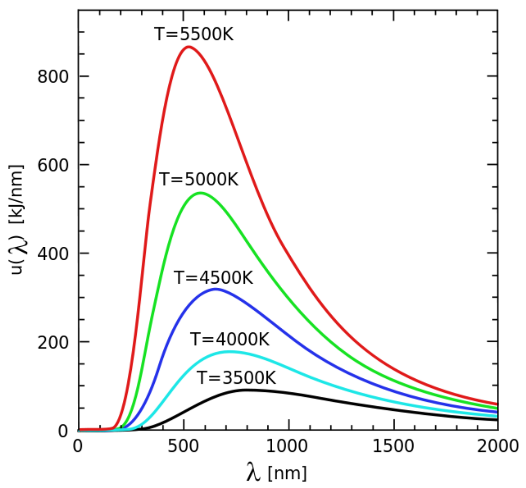
- The main difficulties were in understanding the black body radiation spectrum, photo electricity, Compton effect.

- The other difficulties are in understanding some of the results (at the end of 1800 and beginning of 1900) associated with the development of atomic models, associated with hydrogen spectrum, discovery of X-rays, radio-activities etc.

- For physical interpretation of those vast collection of unexplained experimental data, gathered over a period of more than fifty years, an entirely new concept, which is known today as quantum physics was born.

- Among those atomic and sub-atomic phenomena, some are milestones.

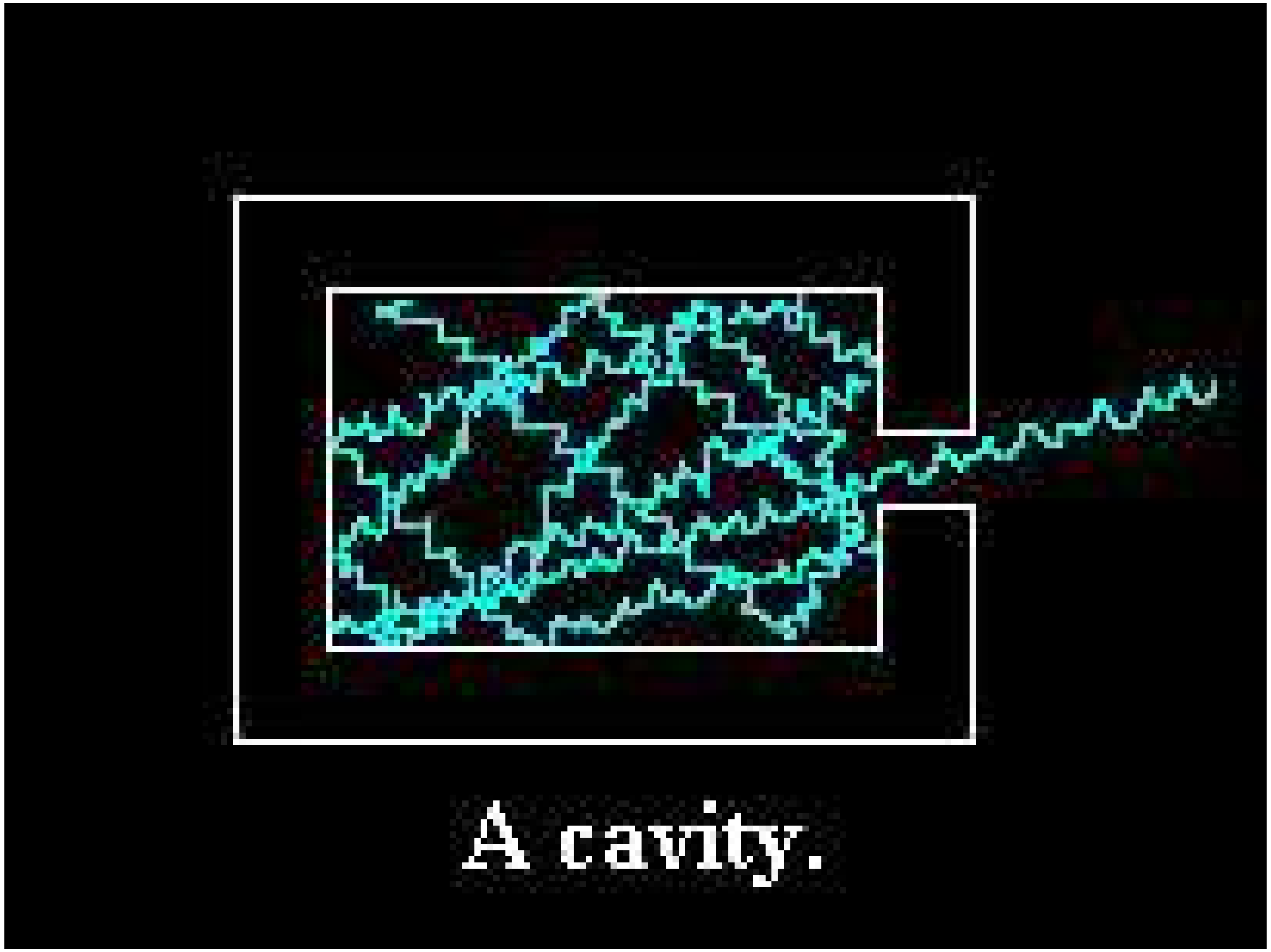
Black Body Radiation Spectrum



A black body refers to an object or system which in principle can absorb and emit radiations of all possible frequencies

Black bodies emit larger quantities of some wavelengths than others depending on the temperature of the body.

Schematic Diagram of a Black-Body Cavity



Experiment:Lummer and Pringsheim (1900)

No ideal black body is possible in nature!

Rayleigh-Jeans Law: Lord Rayleigh and J.M. Jeans (1900).

The classical idea of Rayleigh and Jeans: radiated energy $\equiv$ standing wave or resonant modes of the cavity. The idea could explain the long wavelength region of the spectra.

Wien’s Distribution Law. (1893)

Classical idea: Wien’s distribution law can accurately describe the short wavelength region. Wien considered adiabatic, or slow, expansion of black body cavity containing waves of light in thermal equilibrium. The walls are assumed to be perfectly reflecting.

$$\text{Wien's displacement law} : \lambda_m T = \text{constant}$$

Contribution of Max Planck (1900)

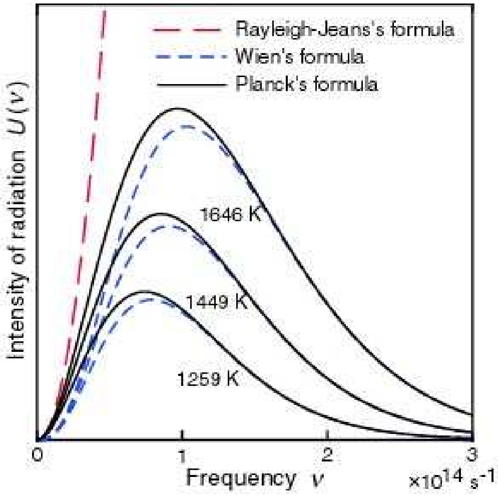
- Max Planck at the age of 42 (in the year 1900) explained the puzzle of **black body radiation**.

- In 1900 Planck proposed that radiation energy is emitted only in definite amount called quanta.

- Historically this is the beginning of Quantum Physics- the so called old Quantum Mechanics or the Quantum Theory.

According to the revolutionary idea of Max Planck: only certain energies can appear and are limited to whole-number multiples of $h\nu$, where ν is the frequency of radiation. Planck originally called h as ”quantum of action” since the product of energy and time is known as action (based on Hamilton’s principle of least action). Today h is known as Planck’s constant and symbolizes the revolutionary new physics.

Comparison: Wien vs Rayleigh-Jeans vs Planck Distribution



Comparison: Planck Distribution vs Experimental Results

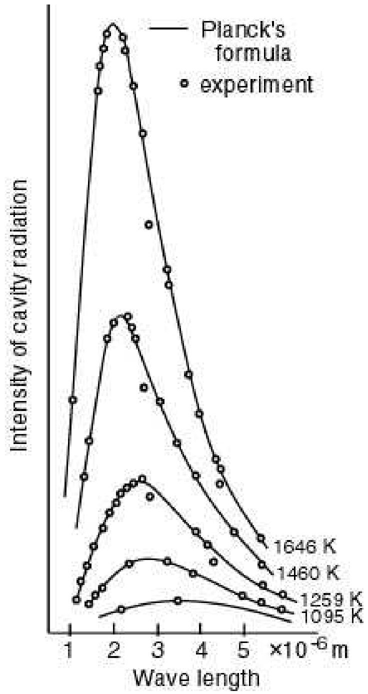
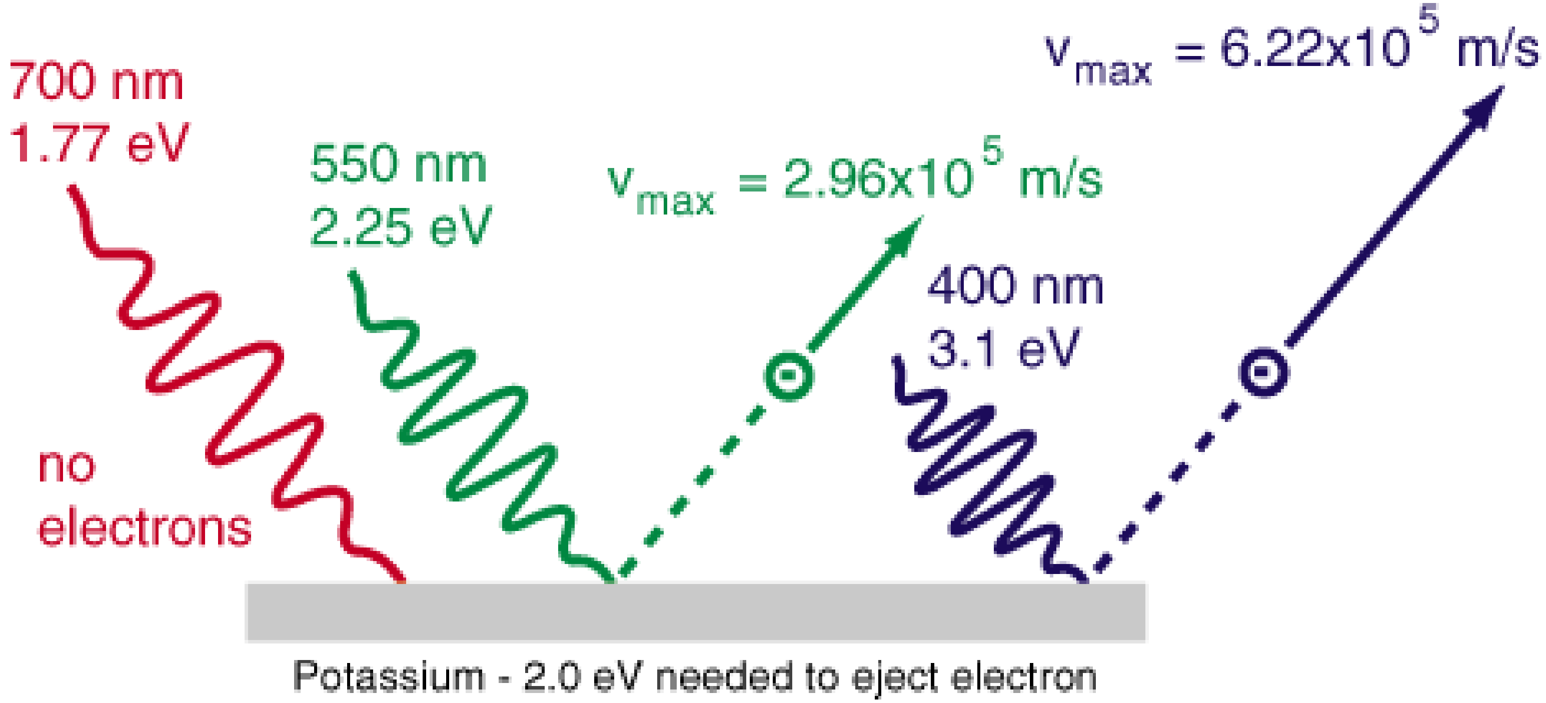


Photo-electric Effect: (1905)

$$E_{photon} = h\nu$$



Photoelectric effect

- The photoelectric effect: emission or ejection of electrons from metals in response to incident light.

- Energy contained within the incident light is absorbed by electrons, giving the electrons sufficient energy to be emitted from the metal surface.

- Classical Idea: Maxwell's EM theory $\longrightarrow$ the more intense the incident light the greater the energy of emitted electrons.

- Lénard's observations:

- (i) Energies of the emitted electrons are independent of the intensity of the incident radiation.

- (ii) A minimum frequency is needed (related to work function of the material) to have electron emission.

- Einstein (1905) successfully resolved this paradox: proposed that the incident light consisted of individual quanta.

- *In his work Einstein introduced the idea that light is atomic in nature. Each atom of light retains its total energy content intact as it moves through space. Amount of energy in a cell of light is proportional to its frequency*

- *Physical Chemist Gilbert N. Lewis gave the name* **PHOTON** *in the year* 1926*.*

- Light interacts with the electrons in the metal like discrete particles.

- For a given frequency ν of the incident radiation, the energy carried: $E = h\nu$, where h is Planck's constant.

- Increasing the intensity of the light corresponded, in Einstein's model, increasing the photon flux, while the energy of each photon remained the same.

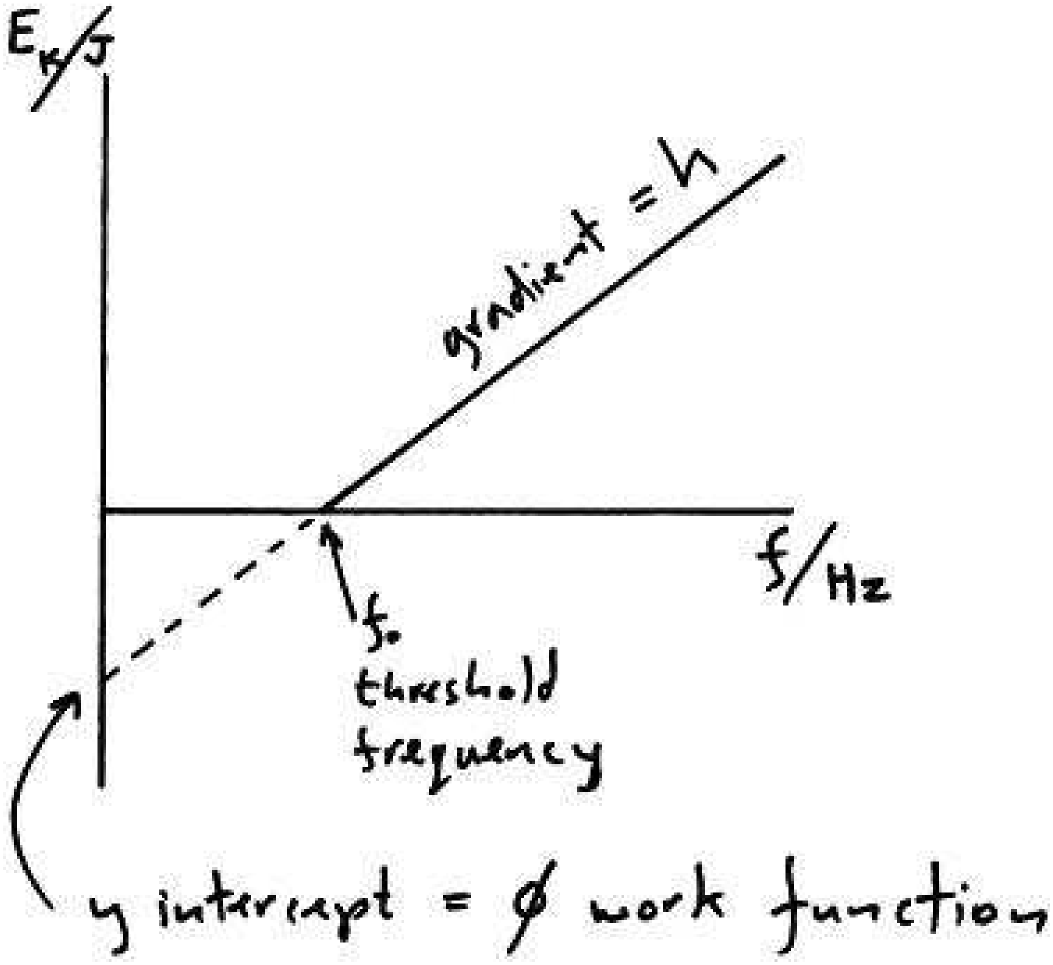
- Increasing the frequency ν, rather than the intensity of the incident radiation would increase the average energy of the emitted electrons.

- These predictions were confirmed experimentally.
- The photoelectric effect is perhaps the most direct and convincing evidence of the discrete nature of light or the quantization of electromagnetic field.
- *Albert Einstein received the Nobel prize in physics in 1921 for explaining the photoelectric effect.*

Einstein's Photo-electric Equation

$$E_k = hf - \phi \text{ where } \phi = hf_0$$

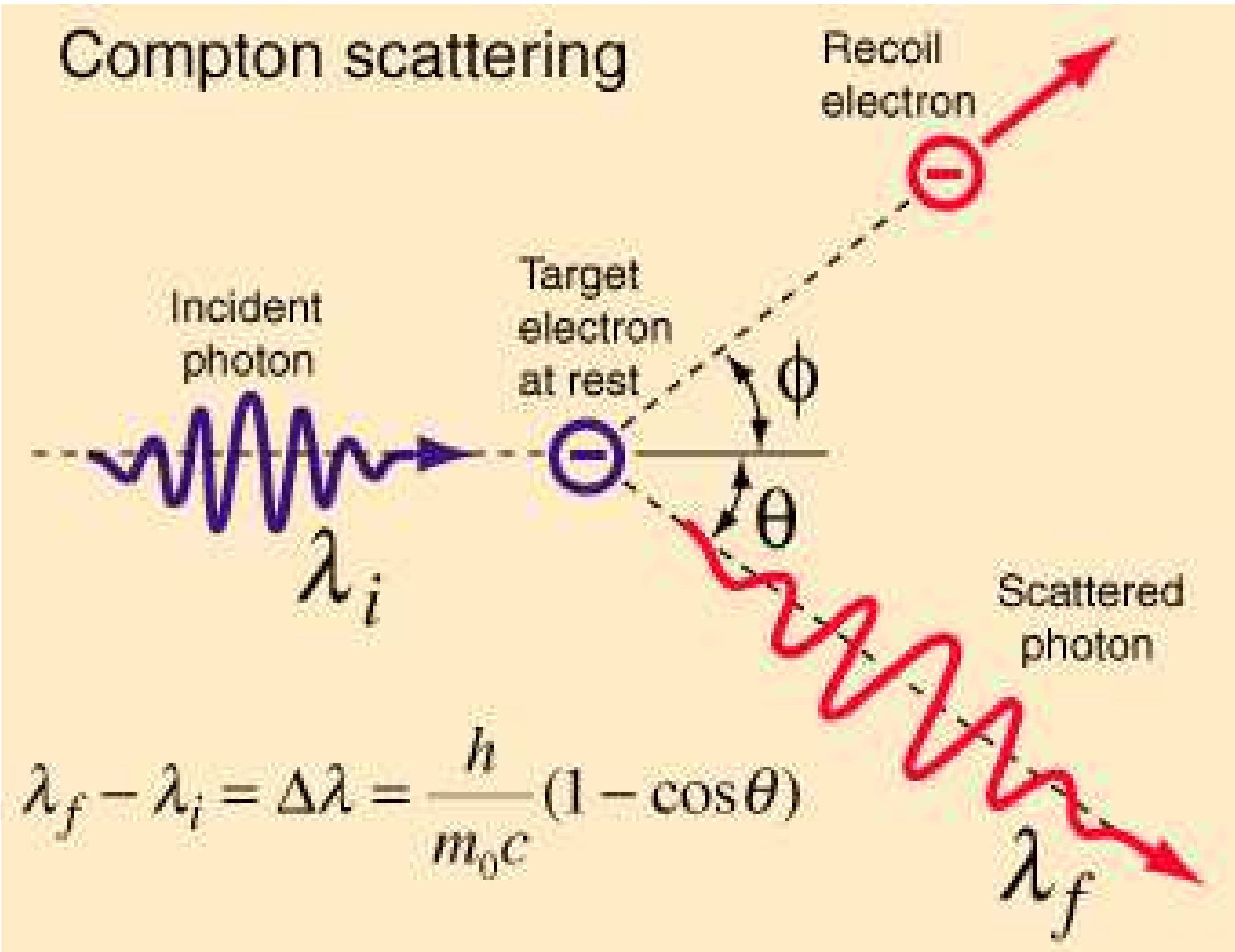
Photo-electric Threshold Frequency and Work Function:



Compton Scattering: (1923)

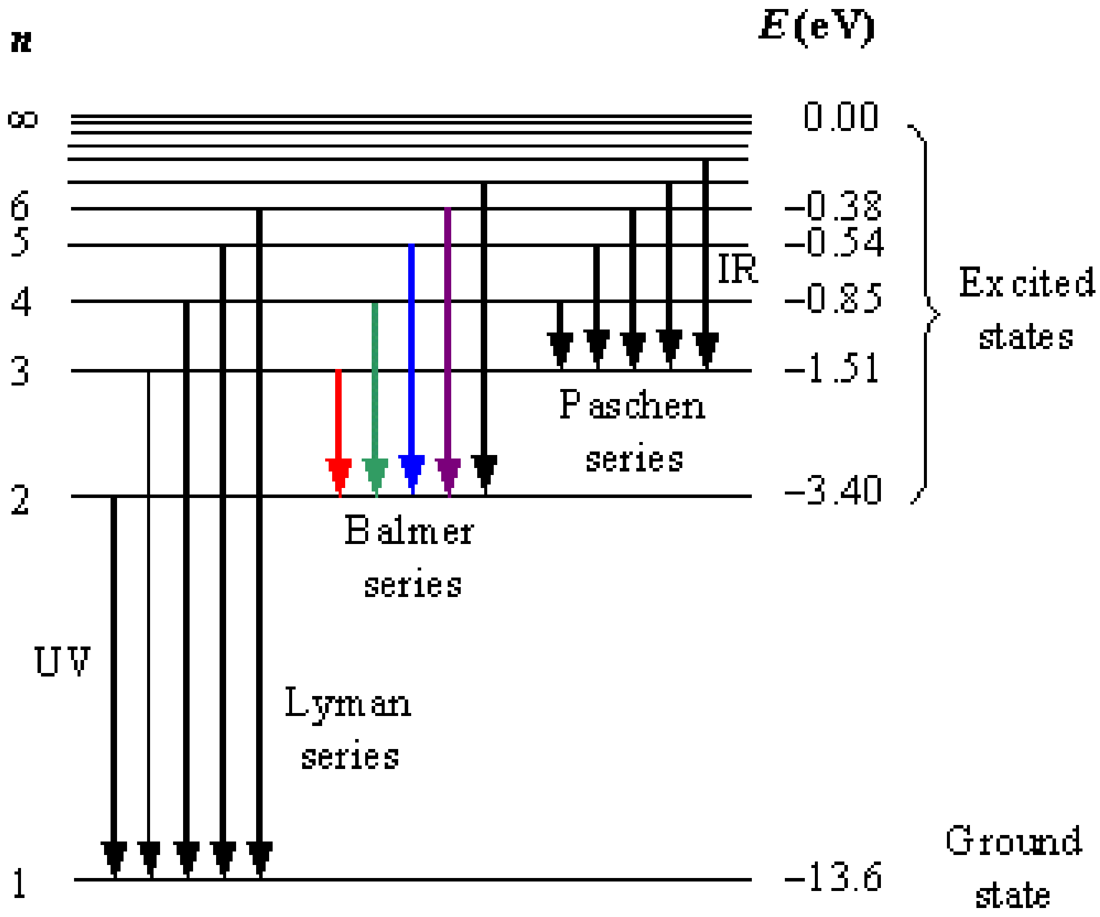
- Compton scattering: scattering of em waves, say X-rays or gamma rays with electrons. The inelastic scattering causes an increase in wavelength of scattered waves $\longrightarrow$ the Compton effect.

- Part of the X-rays or γ-rays energies are transferred to the electrons, $\longrightarrow$ recoil of electrons.

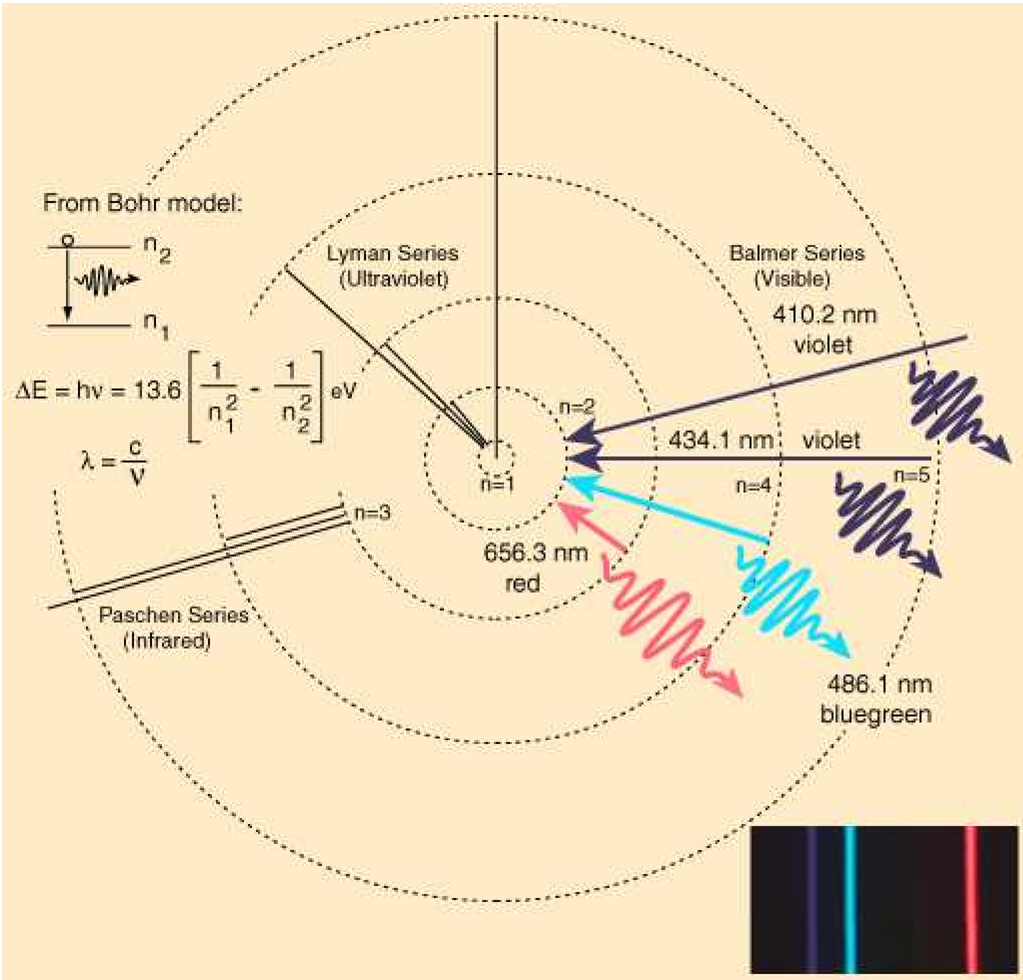

Compton scattering
Recoil
electron
Target
electron
at rest
Incident
photon
ϕ
θ
λ_i
Scattered
photon
λ_f
$\lambda_f - \lambda_i = \Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$

- Compton effect can not be explained unless the particle nature of light is considered. Thomson scattering, the classical theory cannot explain the shift in wavelength of scattered light. Experiment on Compton effect convinced physicists that light can behave as a stream of particle-like objects (quanta) whose energy is proportional to the corresponding frequency.

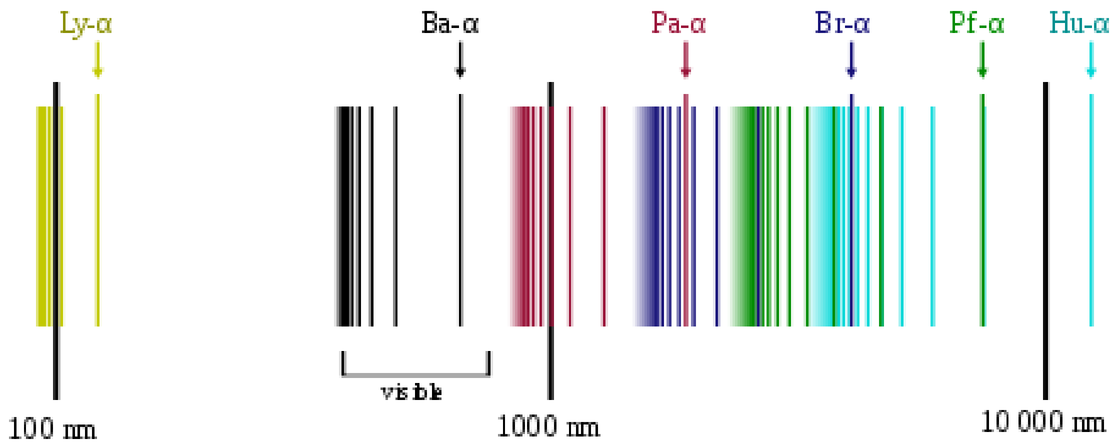
Hydrogen Spectrum

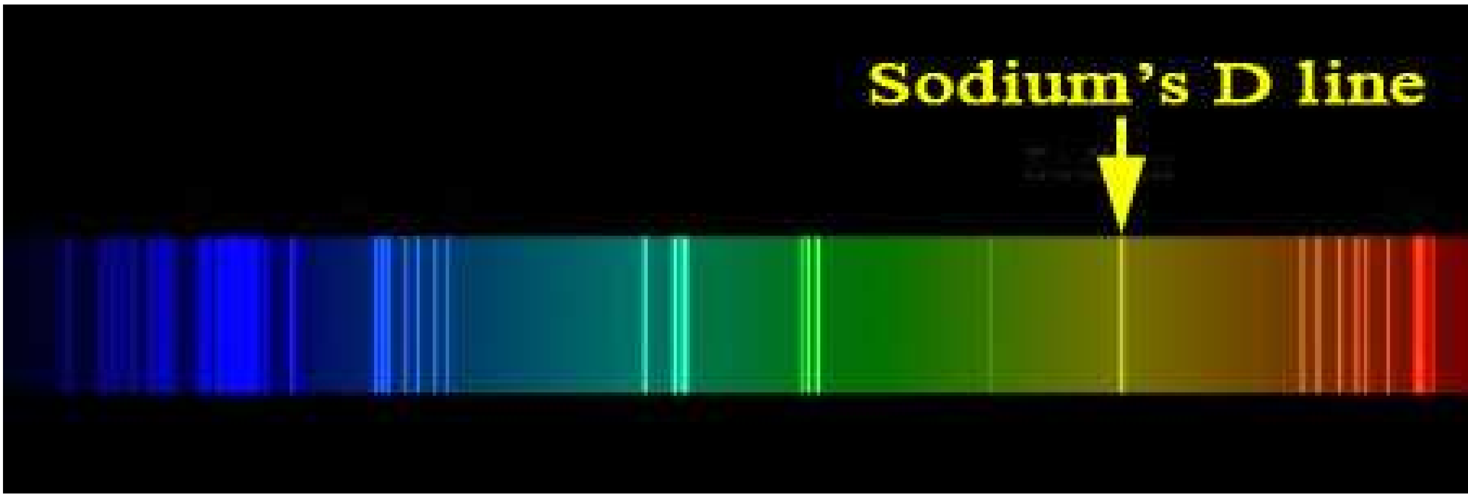


Energy levels of the hydrogen atom with some of the transitions between them that give rise to the spectral lines indicated.



From Bohr model:
n2
n1
ΔE = hν = 13.6 [1/n1² - 1/n2²] eV
λ = c/ν
Lyman Series
(Ultraviolet)
Balmer Series
(Visible)
410.2 nm
violet
434.1 nm
violet
n=2
n=1
n=3
n=4
n=5
656.3 nm
red
486.1 nm
bluegreen
Paschen Series
(Infrared)


Ly-α
Ba-α
Pa-α
Br-α
Pf-α
Hu-α
visible
100 nm
1000 nm
10 000 nm


Sodium's D line

Rydberg's Empirical Formula (1885) for Hydrogen Spectrum:

$$\overline{\nu} = R\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

m and n are integer numbers. m is fixed and n is variable.

$R = 109720\text{cm}^{-1}$ $\longrightarrow$ Rydberg constant.

Classical Idea

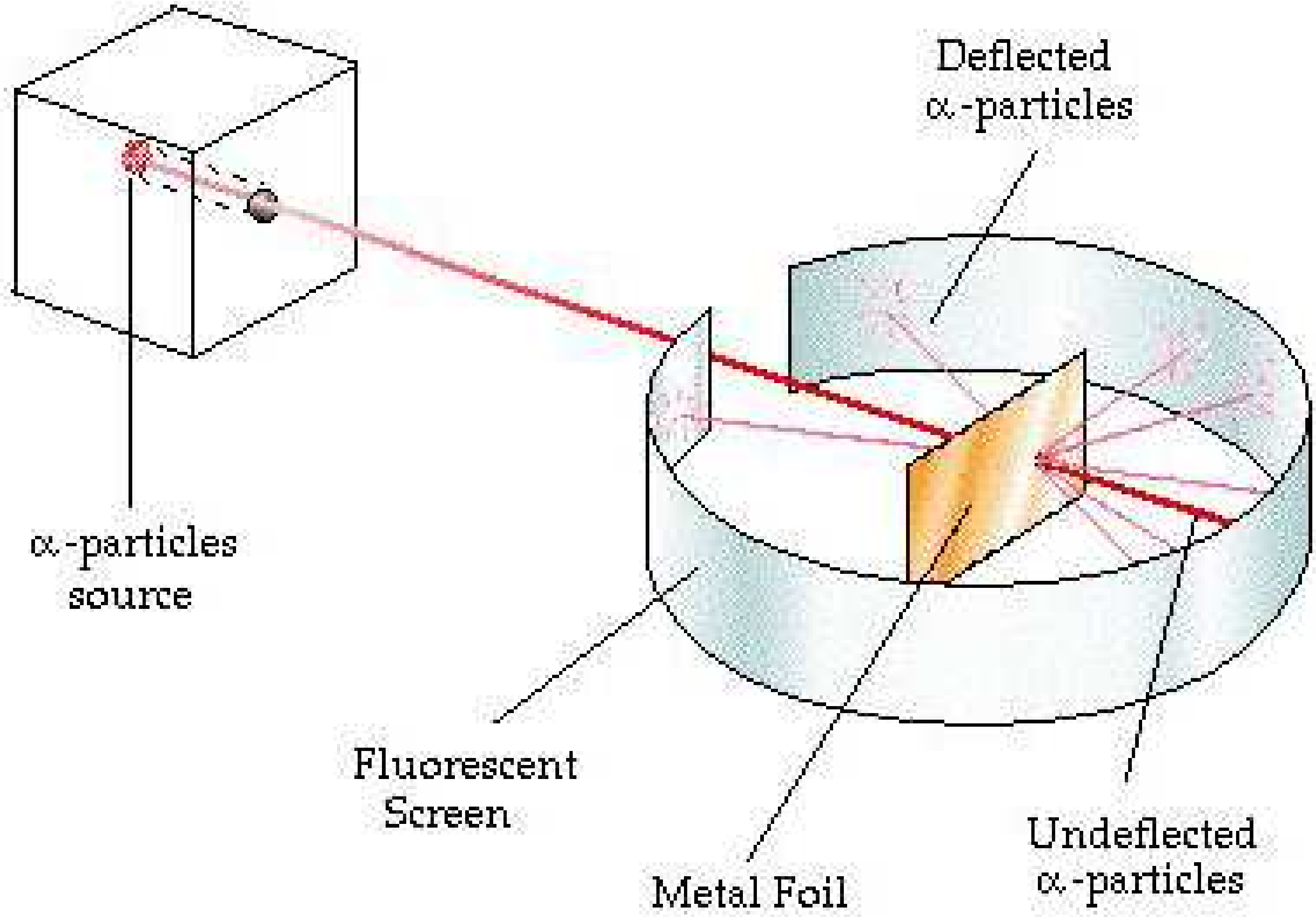
- First real foundation of the modern conception of atoms: Michael Faraday's work on electrolysis $\longrightarrow \pm 1.59 \times 10^{19}$C charge is transferred.

- A more definite idea: Discovery of electron by J.J. Thomson and measurement of Mass and charge of e (1897)

- Thomson's model: electrons are the part of an atom. Atom is charge neutral $\Longrightarrow$ equal quantity of positive charge must be there. Atoms are stable. Unfortunately it can generate only one spectral line.

- Obviously Thomson's model is defective.

Rutherford’s Experiment and Atomic model (1911)



Rutherford Model

- Atoms are hollow objects. Positive charge is at the centre of very tiny size
- Initial assumption: electron is at rest at a definite distance from the nucleus $\longrightarrow$ not a stable configuration.
- Next assumption: electron is rotating about the nucleus $\longrightarrow$ Rotated electrons emit em radiation. They loose energy and fall on the nucleus. Therefore Rutherford model is also not a stable configuration.
- Conclusion: Rutherford model is also defective.

- Immediately after the discovery of electron (1897) it wasn't clear where electrons were located within atoms.

- At the beginning of 1900 the atomic hypothesis was widely but not universally accepted. Atoms were considered point particles, and it wasn't clear how atoms of different elements differed (Lenard 1903).

- One important outstanding problem concerned at that time was the colors emitted by atoms in a discharge tube. No one could understand why different gas atoms glowed in different colors.

Bohr's Model and Bohr's Postulates:

- Niels Bohr was the first to apply the concept of quantum physics to atomic structure. The most impressive result of Bohr theory: explained the appearance of series of lines in hydrogen spectrum.

- Electrons are rotating around the nucleus $\longrightarrow$ Rutherford type model.

- Specific (Privileged) orbits: (a) Stable and Stationary and (b) Non-radiating orbits.

- An atom neither absorbs nor emits radiation continuously but only in finite steps or quantum jumps.

- An atom emits radiation only when it made a transition between states. The frequency of the emitted radiation: $\nu = (E_2 - E_1)/h$.

- Various frequencies of the radiation emitted by an atom are not equal to the frequencies with which the electrons moved within the atom. This was a bold idea that some of Bohr's contemporaries found difficult to accept.

- The significance of the Bohr model is that the laws of classical mechanics apply to the motion of the electron about the nucleus only with some restrictions by quantum rules.

- The angular momentum L is restricted to be an integer multiple of a fixed unit:$\hbar = h/2\pi$ ($L = n\hbar$).

Limitation of Bohr’s Model

- Can solve spectroscopic problem for simplest type atoms- hydrogen or hydrogen type.

- Fine structures can not be explained.

- Spectroscopic and other problems associated with many electron atoms can not be explained.

- Wilson-Sommerfeld Elliptic orbit- can not solve fine structure problem.

- Sommerfeld’s extension to relativistic motion- can explain to some extent the fine structures of spectral lines.

Introduction of Electron Spin

- The Bohr atom model even with Sommerfeld's modifications could not completely explain the complex fine structure of the spectrum of the simplest atom.

- Could not explain the doublet yellow line of sodium.

- Uhlenbeck and Goudsmit put forward, in 1925, the hypothesis of the spinning electrons.

- $\Longrightarrow$ Vector atom model, an extension of Bohr-Sommerfeld model $\longrightarrow$ explain magnetic properties, spectroscopic problems etc., even for many electron atoms.

Electrons in Atoms and Their Quantum Numbers

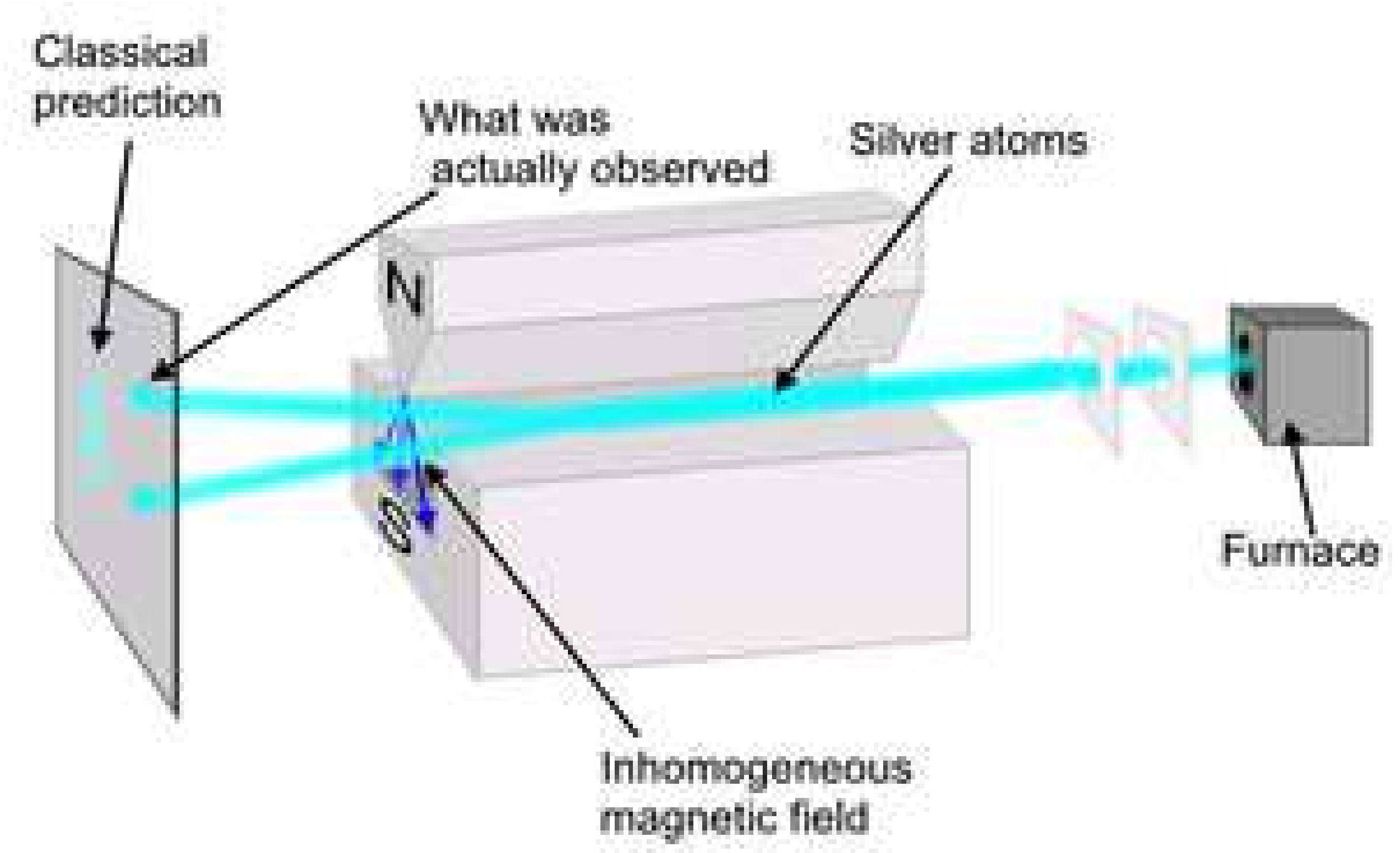
- Principal Quantum Number: $n = 1, 2, \ldots$ positive integers.
- Orbital QN: :$l = 0, 1, 2, ..., n-1$, integers.
- Magnetic Orbital QN: $m_l = -l$ to $+l$, including zero $\Longrightarrow 2l + 1$-number of values.
- Spin QN: $s = \hbar/2$.
- Magnetic Spin QN: $\pm\hbar/2$.

Pauli's Exclusion Principle:

- Every completely defined quantum state in an atom can be occupied by only one electron.

- Equivalently, it is impossible for two electrons in an atom to be identical as regards all their quantum numbers.

- Pauli exclusion principle is satisfied by all elementary particles carrying half-integer spin $\longrightarrow$ Fermions.



Stern-Gerlach Experiment

Raman scattering (1928)

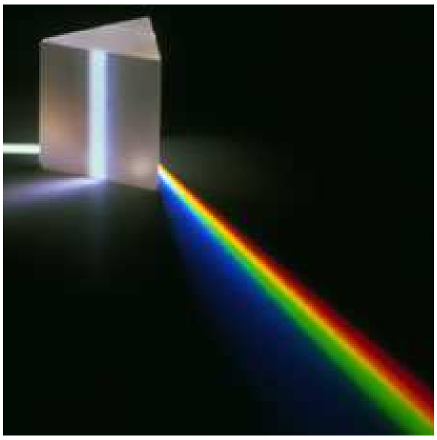
- When light passes through matter, most of the scattered light is of the same wavelength as the incident light. This is known as **Rayleigh scattering** and is considered elastic as the scattered photon has the same energy as the incident photon.

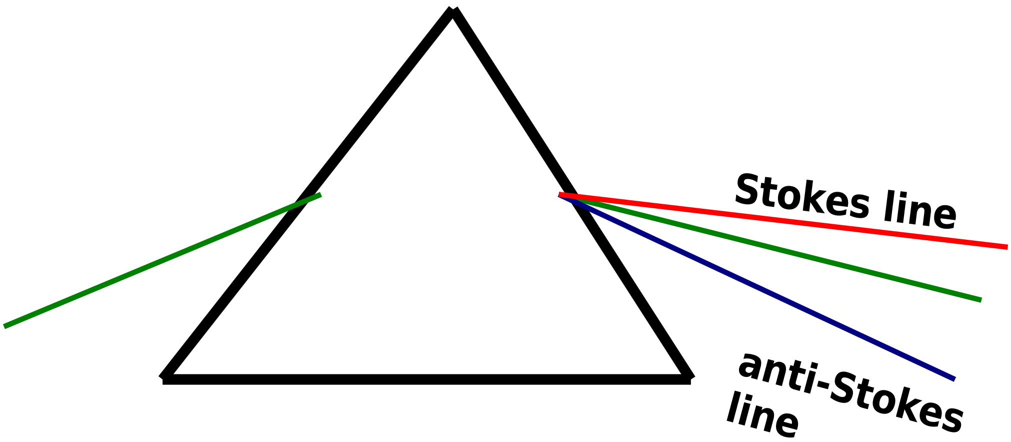
- Some light is inelastically scattered $\longrightarrow$ scattered photon having higher or lower energy than incident one $\longrightarrow$ at a different wave length (or frequency). This is called **Raman Scattering**.

- This shifting of frequency is known as **Raman Effect**, which arises when the incident light excites molecules in the sample, which subsequently scatters the light.

- Raman effect is the appearance of additional lines in the spectrum of monochromatic light that has been scattered by a transparent material medium. Raman spectrum is characteristic of the transmitting substance.

- In the year 1923, Smekal gave a theoretical prediction of what Raman found experimentally during 1928- the Smekel-Raman effect.




Stokes line
anti-Stokes line

- Typically, only one part in a thousand of the total intensity of incident light is Rayleigh scattered, while for Raman scattering this value drops to one part in a million (0.0001 percent).

- The Raman Effect can only be explained if it is assumed that light is made up of particles known as photons.

- *It is said that in 1930, anticipating the Nobel Prize, C.V. Raman bought two tickets to Stockholm, one for himself and the other for his wife, as early as September though the prizes were to be announced formally only in November. Indeed, his conviction was not misplaced; he was decorated with the Nobel Prize for Physics that year for his path-breaking work and the discovery of the Raman effect. The first 'Indian Nobel' for Physics is thus more than 80 years old. And Raman remains the only Indian to receive a Nobel Prize in Physics. Prof. Subrahmanyan Chandrasekhar, won Nobel Prizes, but during that time he was an U.S. citizens.*

- The quantum theory of the period between Planck's proposition of quantum (1900) and the advent of a full-blown quantum mechanics (around 1925-30) is often referred to as the old quantum theory

- Quantum theory: conventional results from Classical mechanics were assumed to hold, but with the additional assumption that only certain values of a physical quantities were allowed $\longrightarrow$ quantized values $\longrightarrow$ sometime called zeroth quantization.

Matter Wave Hypothesis of Louis de Broglie (1924)

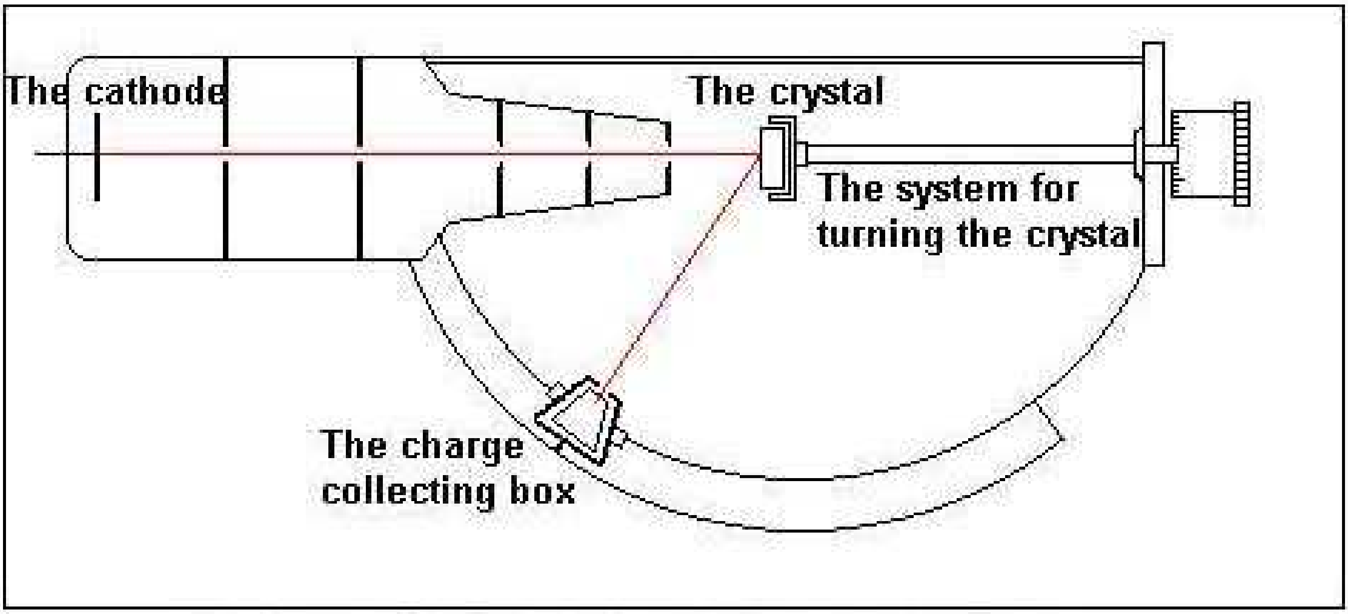
- As an undergraduate de Broglie studied medieval history. During World War I he served in a field radio communication unit and this changed his interest. After the war de Broglie did his doctoral thesis in what has become known as de Broglie waves.

- In 1924 de Broglie speculated that nature did not single out light as the only entity to exhibit wave-particle duality. He proposed that ordinary particles such as electrons could also exhibit wave characteristics in certain circumstances.

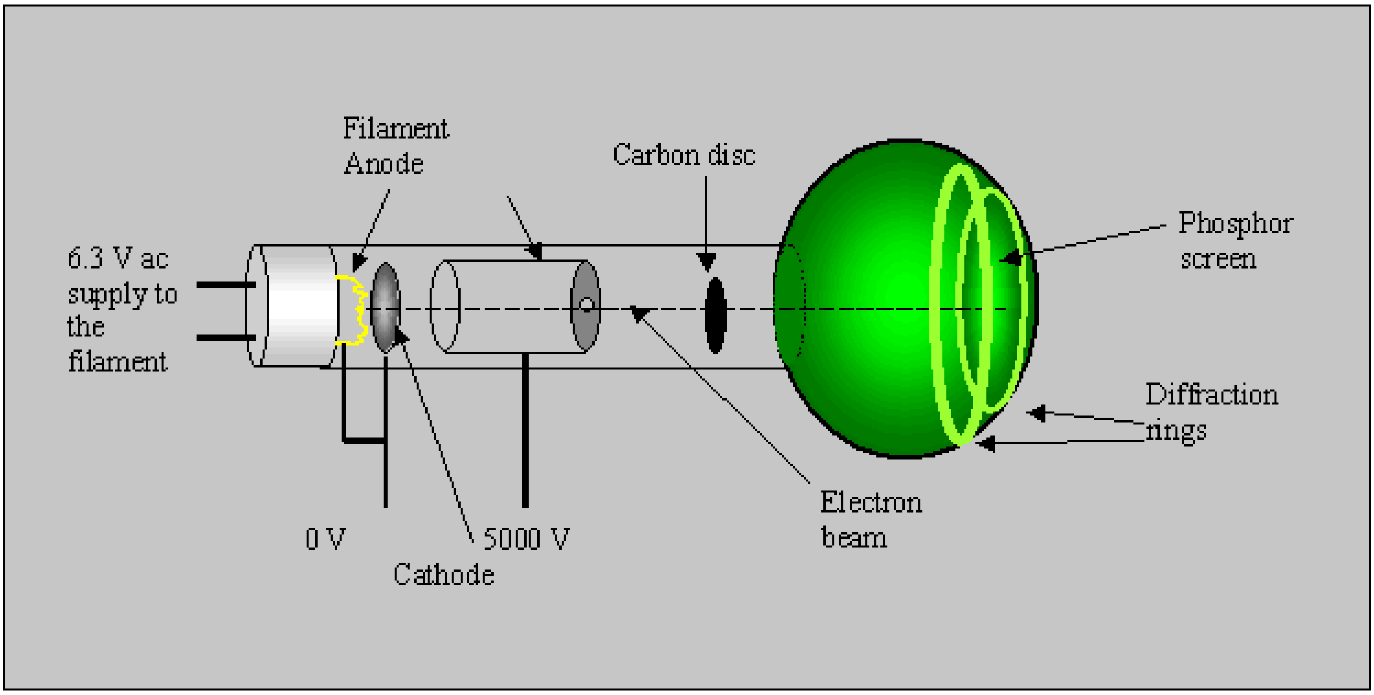
- de Broglie assumed that an electron has associated with it a system of matter waves.

- The distance between successive crests is the de Broglie wavelength, $\lambda = h/p$, where h is Planck's constant and p is momentum.

- In 1926 Davisson and Germer actually saw electron waves demonstrating an interference pattern and de Broglie got Nobel prize.



The diagram for the Davisson-Germer experiment.



Filament
Anode
Carbon disc
6.3 V ac supply to the filament
Phosphor screen
Diffraction rings
Electron beam
0 V
5000 V
Cathode

Wave Mechanics

- Adopting the proposal by de Broglie that particles of matter have dual nature and in some situations act like waves, Schrödinger (1926) wrote the basic equation of quantum mechanics. The Schrödinger equation treats electrons and other elementary particle as waves.

- This formulation with the central idea from de Broglie's hypothesis is called Wave Mechanics.

- The only problem with Schrödinger's equation was that the interpretation of the matter wave was wrong. He described that ψ is the density distribution– e.g., some regions rich in electron matter while others scarce.

- Max Born (1928) figured out that the equation actually predicts the probability of existence of a quantum mechanical system at a particular point.

- The quantity $| \psi |^2$ is associated with the probability density.

- *Quantum Mechanics* $\Longrightarrow$ *Probabilistic; Classical Mechanics* $\Longrightarrow$ *Purely Deterministic.*

Schrödinger Equation:

In 3-D:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi, \qquad \frac{\partial^2\psi}{\partial t^2} = v^2\nabla^2\psi, \qquad \frac{\partial\psi}{\partial t} = D\nabla^2\psi$$

In 1-D:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}, \qquad \frac{\partial^2\psi}{\partial t^2} = v^2\frac{\partial^2\psi}{\partial x^2}, \qquad \frac{\partial\psi}{\partial t} = D\frac{\partial^2\psi}{\partial x^2}$$

Interestingly, Schrödinger Equation is not a wave equation. It is a diffusion equation in complex space time geometry. Newton's eqn.:

$$m\frac{d^2x}{dt^2} = F: \quad 1-D, \quad m\frac{d^2\vec{r}}{dt^2} = \vec{F}: \quad 3-D$$

In general:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r},t) + V(\vec{r})\psi(\vec{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t)$$

Formal Structure:

$H\psi = E\psi$, (Eigen value equation), $H \longrightarrow$ Hamiltonian operator.

- Schrödinger equation $\longrightarrow$ diffusion equation in complex Euclidean space, called Hilbert Space.

- Hilbert Space (end of the 19th century:)

- David Hilbert was one of the outstanding mathematicians of the modern era. Hilbert's work on infinite-dimensional space, later called Hilbert space, proved invaluable for quantum mechanics.

- Today quantum mechanics is said to be a theory set in Hilbert Space.

- The mathematical concept of a Hilbert space, generalizes the notion of Euclidean space.

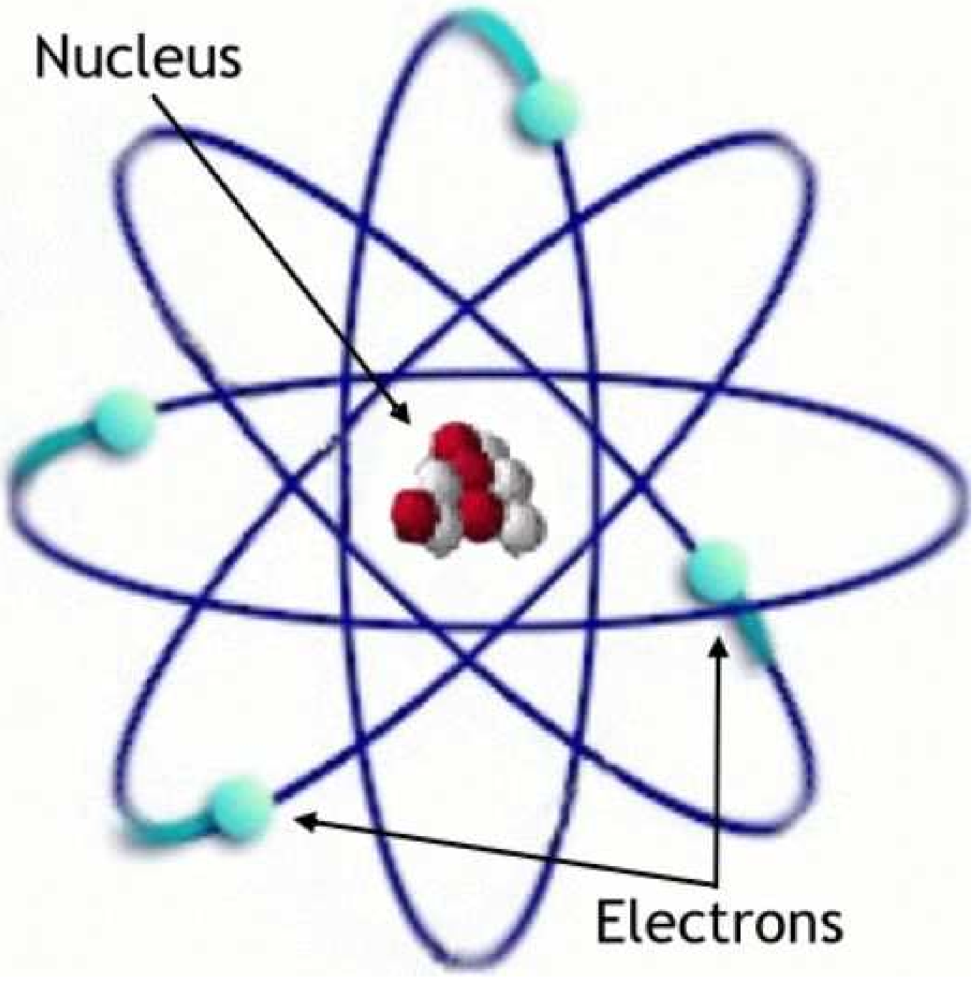
- It extends the methods of vector algebra and calculus from the two-dimensional plane and three-dimensional space to spaces with any finite or infinite number of dimensions.

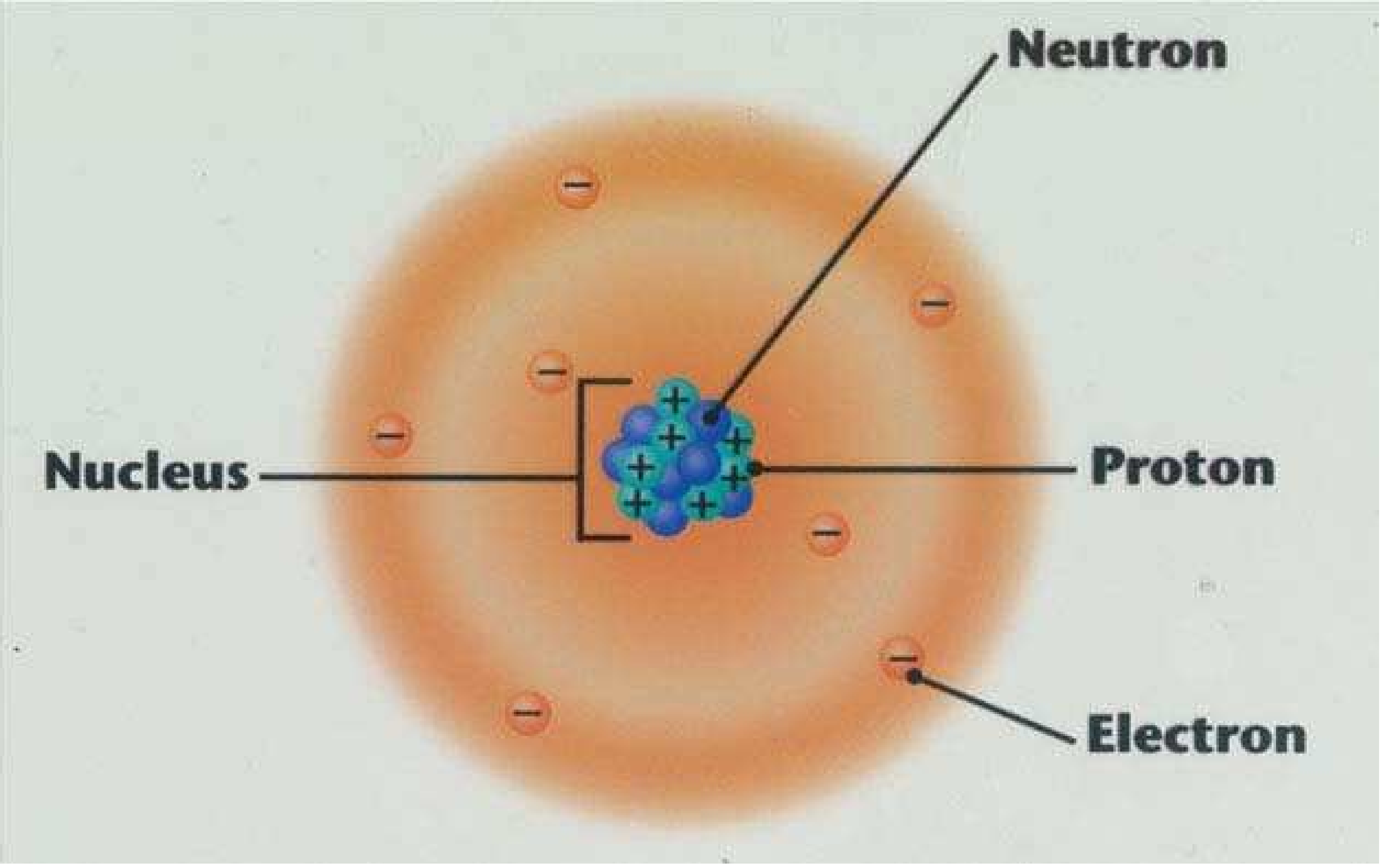
- The distinction between the two interpretations is important. If $|\psi|^2$ is small at a particular position, the original interpretation implies that a small fraction of an electron will always be detected there.

- In Born's interpretation, nothing will be detected there most of the time, but when something is observed, it will be a whole electron (collapse).

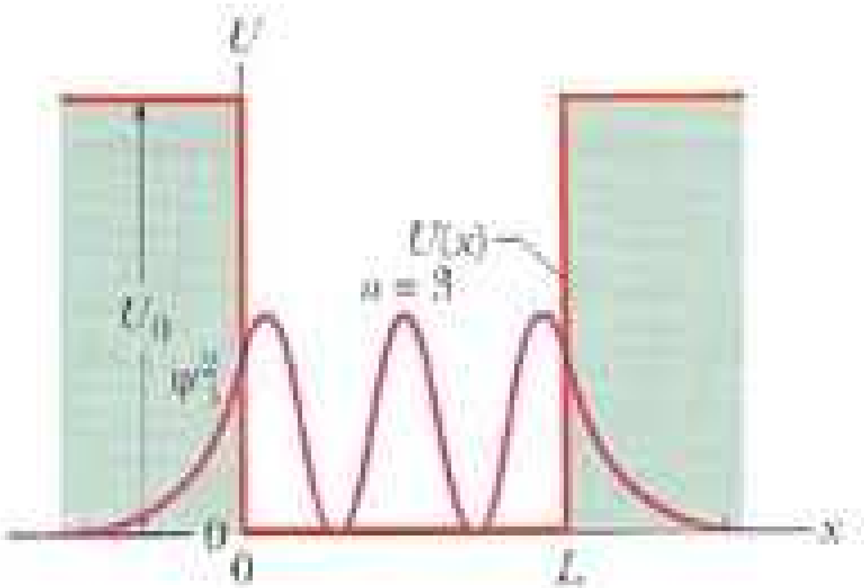
- The concept of the electron as a point particle moving in a well-defined path around the nucleus is replaced in wave mechanics by clouds that describe the probable locations of electrons in different states.

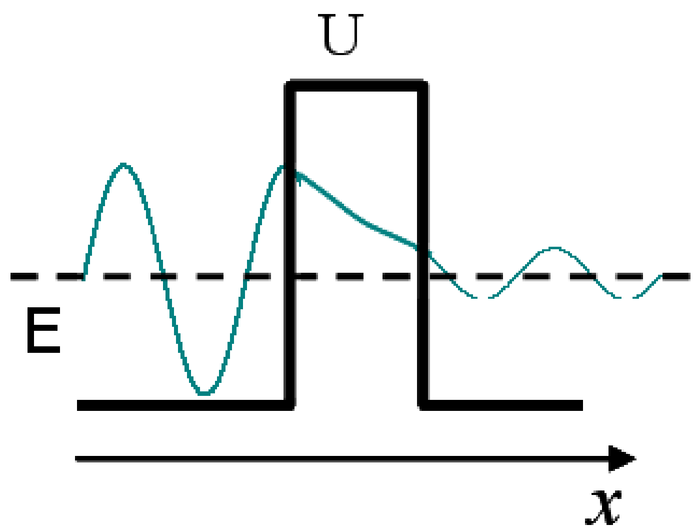
- Born's probability density is perhaps the most dramatic change in viewing our world since Newton and his gravity.


Nucleus
Electrons

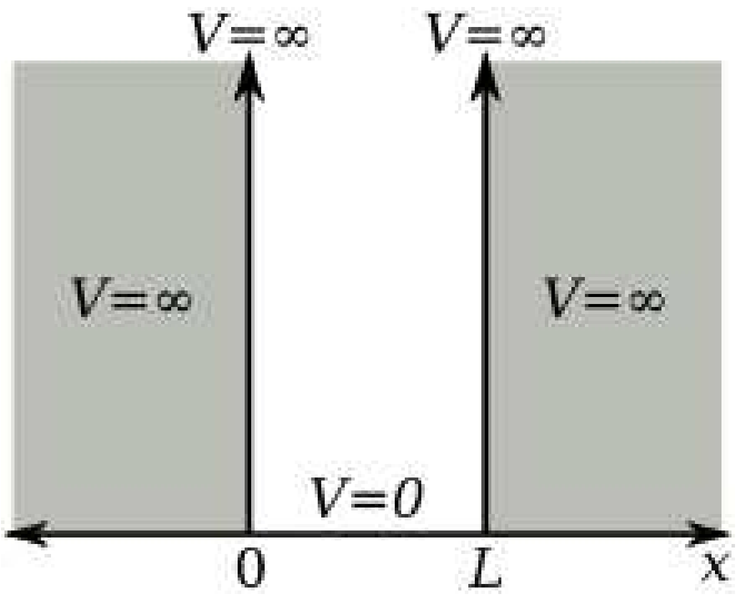

Neutron
Nucleus
Proton
Electron

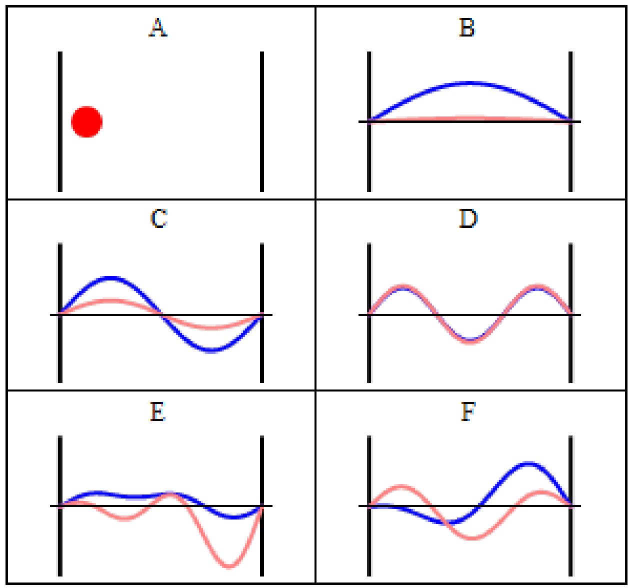
Potential well and Barrier penetration- having no classical explanation





Infinite Potential well





First Quantization

- Dynamical variables of Classical Mechanics (positional coordinates, linear and angular momenta, Hamiltonian, etc.), $\longrightarrow$ Operators in Quantum Physics.

- Canonical Quantization: $xp_x - p_x x = i\hbar$, ... Angular momentum vector $\vec{L}$: $\vec{L} \times \vec{L} = i\hbar\vec{L}$.

Matrix Mechanics- Heisenberg

- At just about the same time as the announcement of Schrödinger equation, came that of another theory, called Matrix Mechanics due to Heisenberg.

- Despite the vastly different appearance of the two theories it was very soon recognized that they are completely equivalent.

- Dynamical variables $\longrightarrow$ matrices.

- Working essentially independently, in the mid-1920's Heisenberg and Schrödinger both created a full form of Quantum Mechanics.

- Making use of matrix algebra, Heisenberg (1925) developed matrix mechanics form of quantum mechanics.

- The matrix mechanics that Heisenberg built is discrete in nature.

- The consequence of Heisenberg's work is his revolutionary (1927) Uncertainty Principle: $dq dp \geq h$.

- Position and momentum of a quantum mechanical system can not be measured simultaneously.

- $dq \longrightarrow$ uncertainty in q, similarly dp is for momentum uncertainty.

- Uncertainty principle is consistent with de Broglie's hypothesis, Schrödinger equation and canonical quantization rule.

Spin in Quantum Mechanics

- Spin is introduced by hand in Schrödinger equation. It was done by Pauli.

- Form of Spin wave functions:

$$\vec{s} = \frac{1}{2}\vec{\sigma}\hbar \text{ where } \vec{\sigma} \longrightarrow \text{ Pauli spin matrices}$$

- Where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ and } \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Spinors:

- $$\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ up spin state and } \beta = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ down spin state}$$

- $$\sigma_z \alpha = +1\alpha \text{ and } \sigma_z \beta = -1\beta$$

Relativistic Quantum Mechanics

- Combination of special theory of relativity and quantum mechanics:

$$\left[\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right) - \left(\frac{mc}{\hbar}\right)^2\right]\psi(t,\vec{r}) = 0$$

$\Longrightarrow$ Klein-Gordon equation (Oskar Klein and Walter Gordon, 1926)- Later development by Schrödinger.

- Problems with KG-equation:
(i) Possibility of negative energy for free particles $[E = \pm(p^2c^2 + m^2c^4)^{1/2}]$ $\longrightarrow$ unphysical and (ii) Negative probability density $\longrightarrow$ is also unphysical.

Dirac Equation (1928):

- Dirac intuitively proposed a new relativistic quantum mechanical equation.

- Known as Dirac Equation:

$$(\vec{\alpha}.\vec{p} + \beta mc)\psi(\vec{r}, t) = i\hbar\frac{\partial\psi}{\partial t}$$

  Here $\vec{\alpha}$ and β are 4×4-matrices.

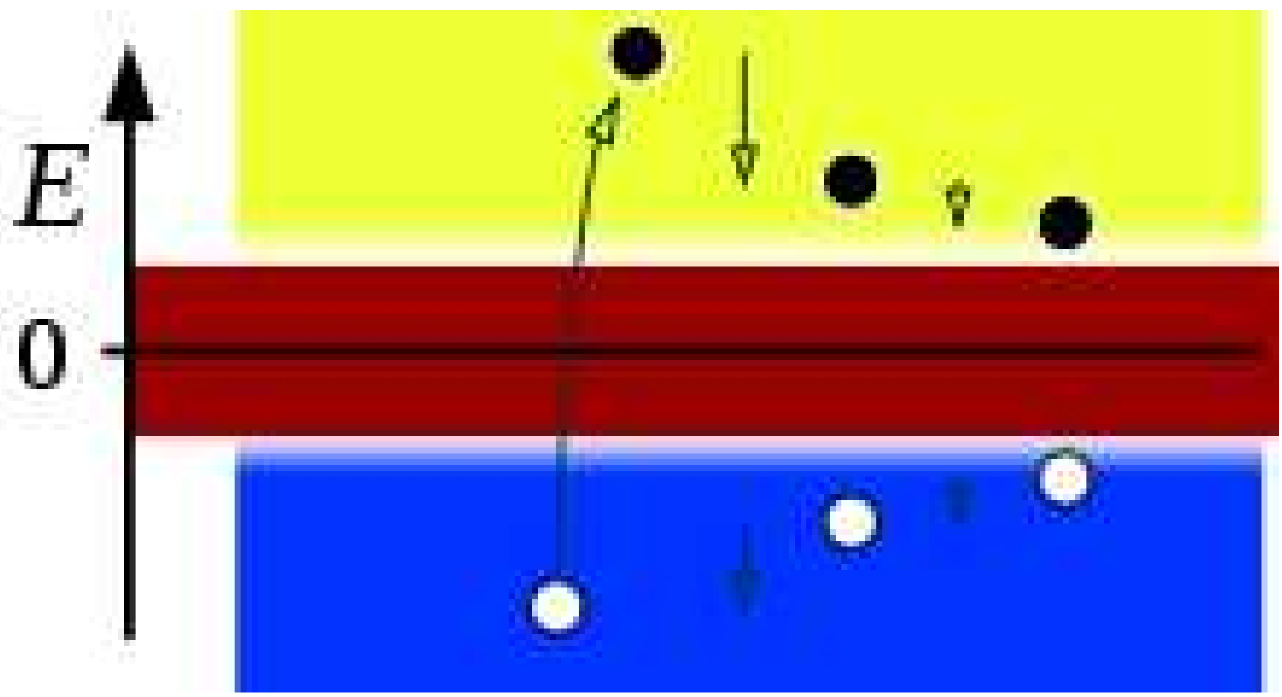
$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \text{ where } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

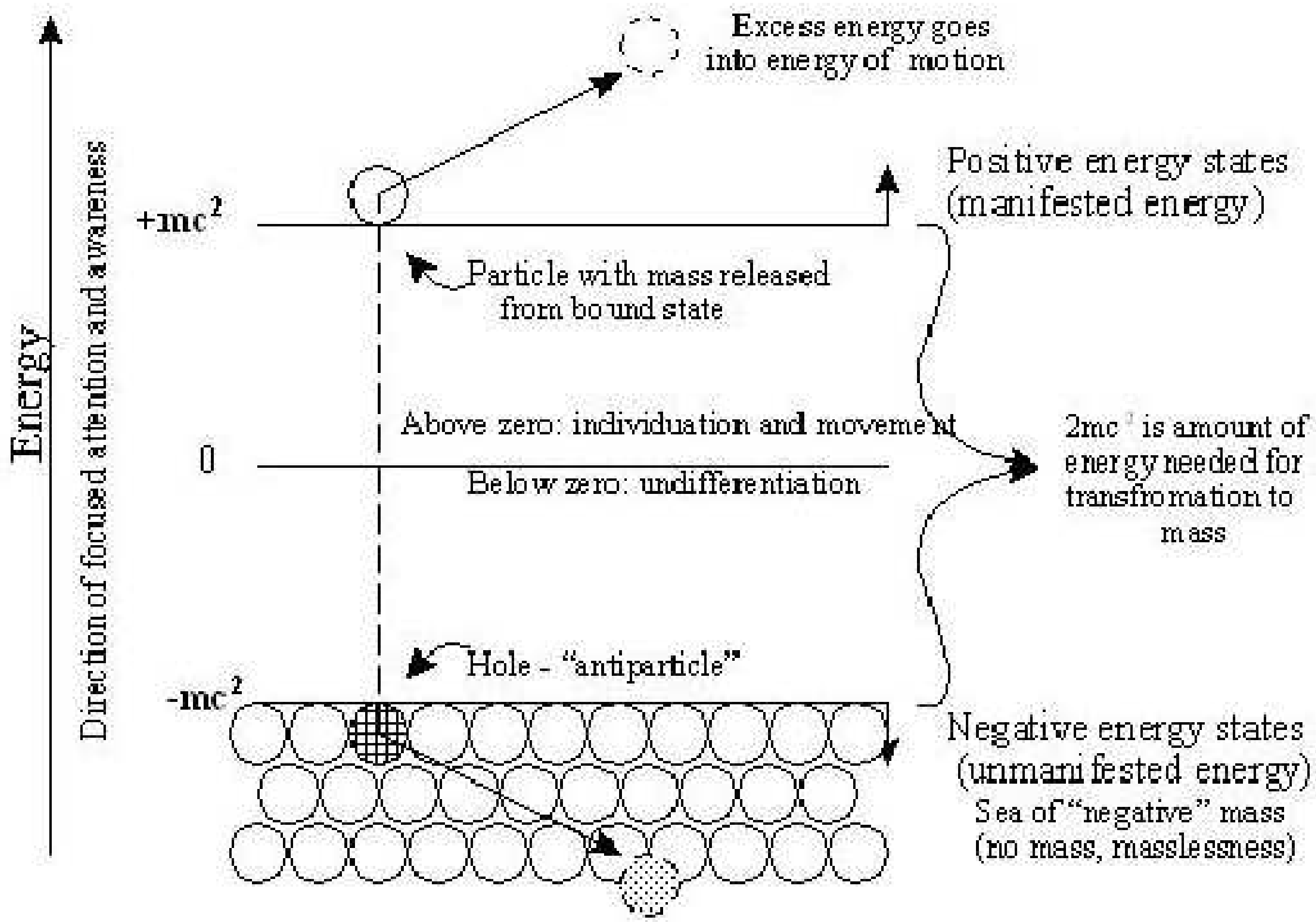
- Wave function ψ is a column matrix (1×4).

- Negative probability problem is solved.

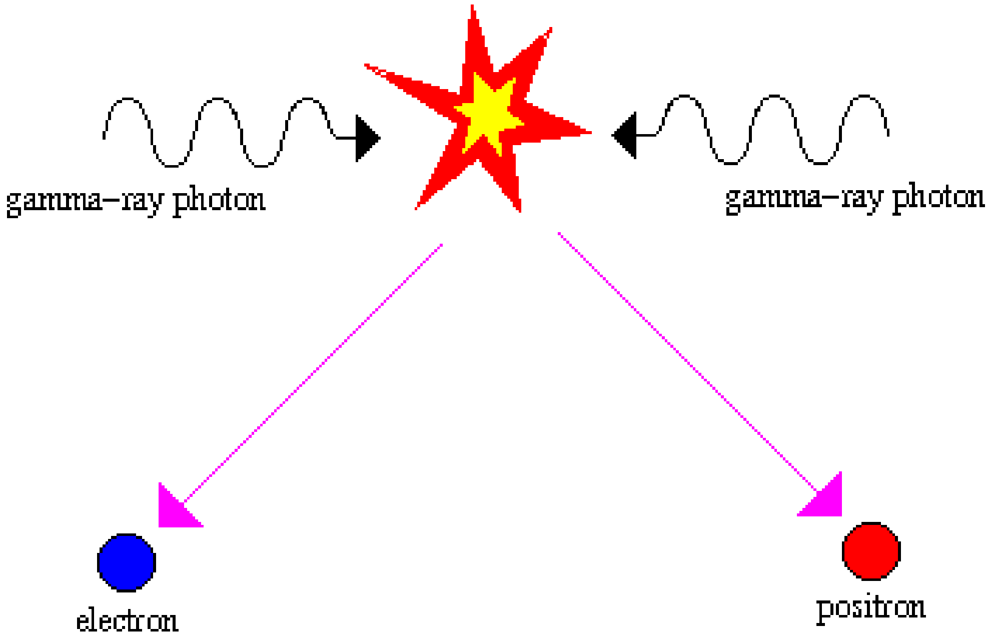
- Problem of negative single particle energy is still there.

- Infinite negative energy Dirac-sea as a possible solution.


E
0


Excess energy goes into energy of motion
Positive energy states (manifested energy)
+mc²
Particle with mass released from bound state
Energy
Direction of focused attention and awareness
Above zero: individuation and movement
0
Below zero: undifferentiation
2mc² is amount of energy needed for transfromation to mass
Hole - "antiparticle"
-mc²
Negative energy states (unmanifested energy)
Sea of "negative" mass (no mass, masslessness)

Pair Production



when two high energy gamma−ray photons collide, an electron/positron pair are produced (the energy is converted into mass, $E=mc^2$)

- Pauli exclusion principle + Dirac sea $\longrightarrow$ solves negative energy problem for fermions.

- KG-equation satisfied by bosons. Do not obey Pauli exclusion principle.

- Dirac sea is also called Fermi sea.

- No Bose sea.

- Negative energy problem for bosons can not be solved.

- Probability density $\equiv$ charge density $\longrightarrow$ can be positive or negative.

- Except the minor RADIATIVE CORRECTION part, complete hydrogen spectrum can be explained if the electron in hydrogen atom is assumed to obey Dirac equation in presence of Coulomb interaction with the nucleus.

Some Remarks on Relativistic Quantum Mechanical Equations and Related Physics

- KG-equation: (i) negative energy and (ii) negative probability density.

- Dirac Equation: (i) negative probability problem is solved. (ii) Solution of negative energy problem $\longrightarrow$ help of multi-particle system.

- Dirac sea $\longrightarrow$ Fermions. No solution for Bosons.

$\downarrow\downarrow\downarrow$

Starting Point of a completely new subject: Many body Quantum Physics (Mechanics) or Quantum Field Theory (1950)- Dirac, Fock, Pauli, Heisenberg, Bogolyubov

Many Particle System in Classical Mechanics

- New physical quantity: Lagrangian $\longrightarrow L = T - V$

- Minimization of action

$$\begin{aligned}\delta S &= \delta \int_1^2 L dt = 0 \equiv \\ &\text{Fermat's principle : Extremization of OPTICAL PATH :} \\ &\delta \sum_i \mu_i \Delta l_i = 0 \\ &\delta \int_1^2 \mu(l) dl = 0 \text{ continuous case}\end{aligned}$$

Euler-Lagrange equation:

- N-number of particle, 3-degrees of freedom for each: Generalized coordinates $\longrightarrow \{q_i(t)\},\ i = 1, 2, ...3N$. Generalized velocity $\longrightarrow \{\dot{q}_i(t)\},\ i = 1, 2, ...3N$, $3N$-degrees of freedom.

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i}$$

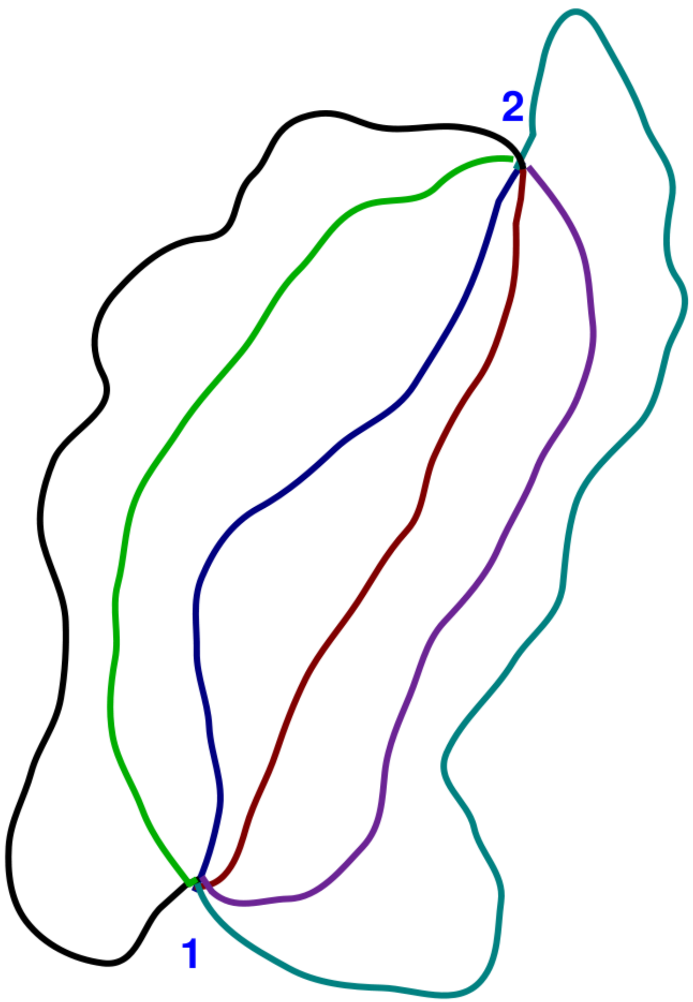
- Similar idea in Quantum Field Theory:

- Action:

$S = \int_1^2 Ldt = \int dtd^3r\mathcal{L}$ Lorentz invariant $d^4x = d^3rdt \Longrightarrow \ S = \int \mathcal{L}d^4x$

- Minimization of action: $\delta S = 0$

Path between two space time points

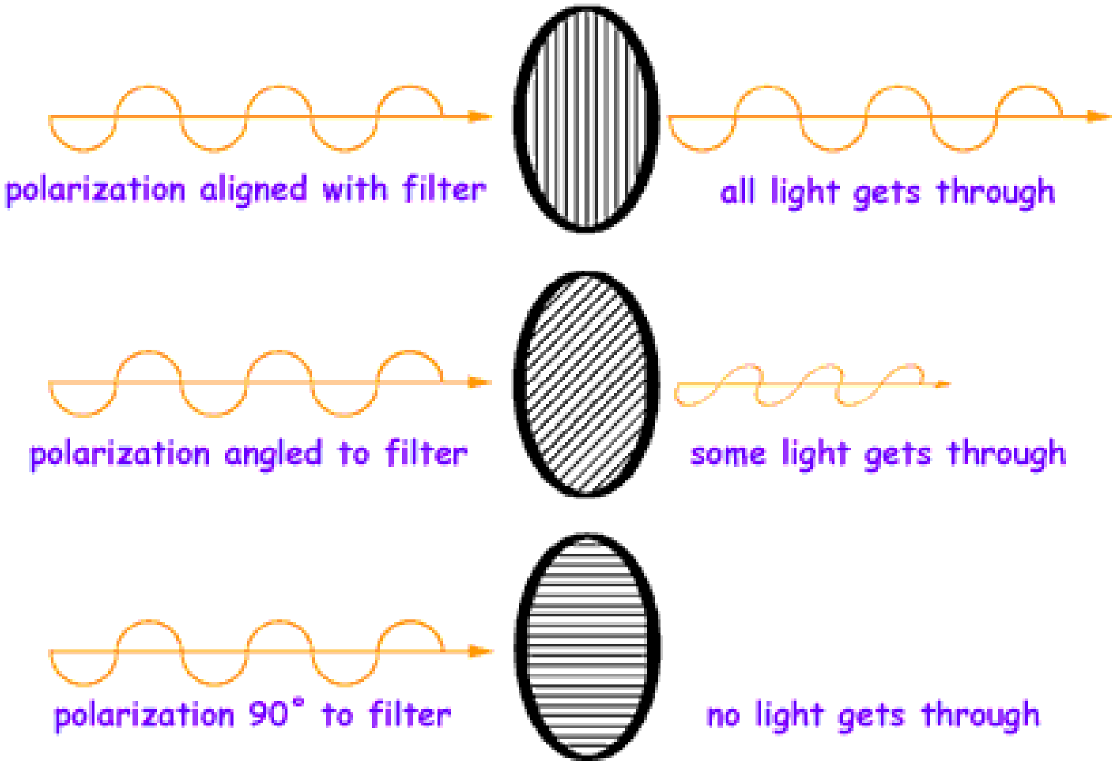


Second Quantization

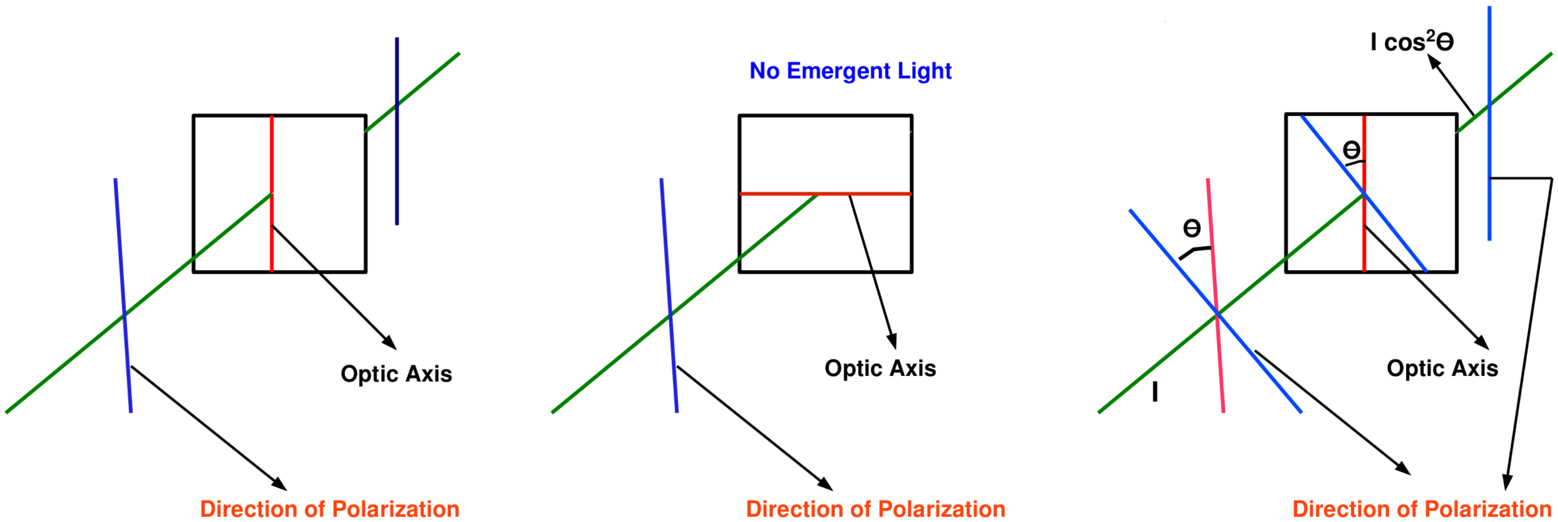
- Wave functions $\longrightarrow$ Field Operators.
- Field Operator: $\psi(t, \vec{r}) \equiv \psi(x) \equiv \psi_{\vec{r}}(t)$
- Infinite number of degrees of freedom.
- $\Longrightarrow$ Relativistic version of Euler-Lagrange eqn.
- Particle in many body quantum system $\longrightarrow$ Field Quanta: Photons, Electrons etc.

Wave-Particle Duality

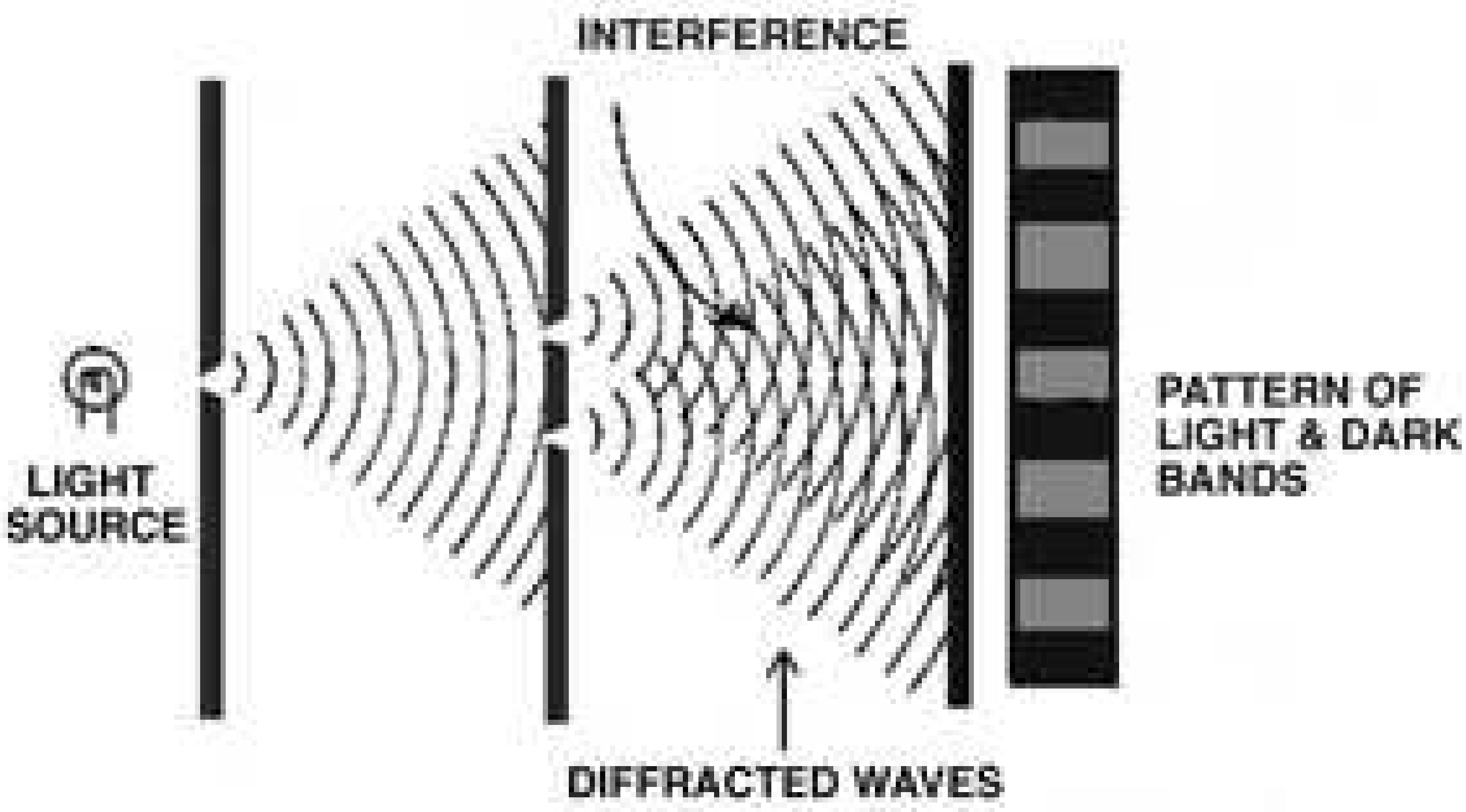
Polarization of Light



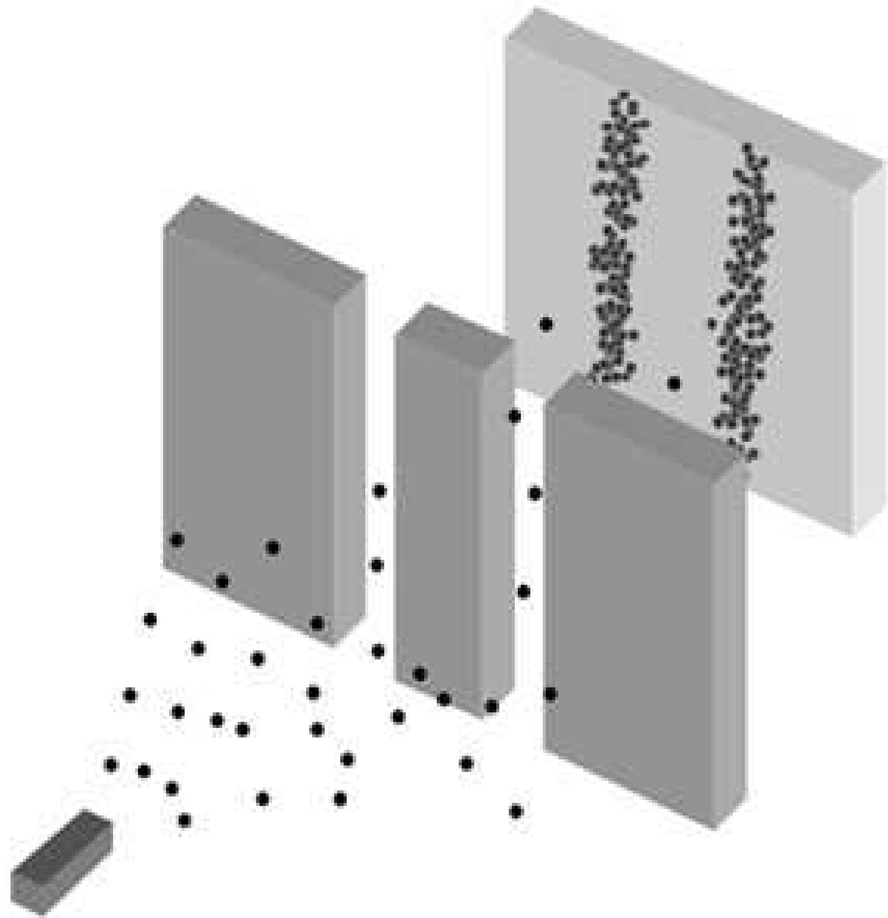
Polarization of Single Photon



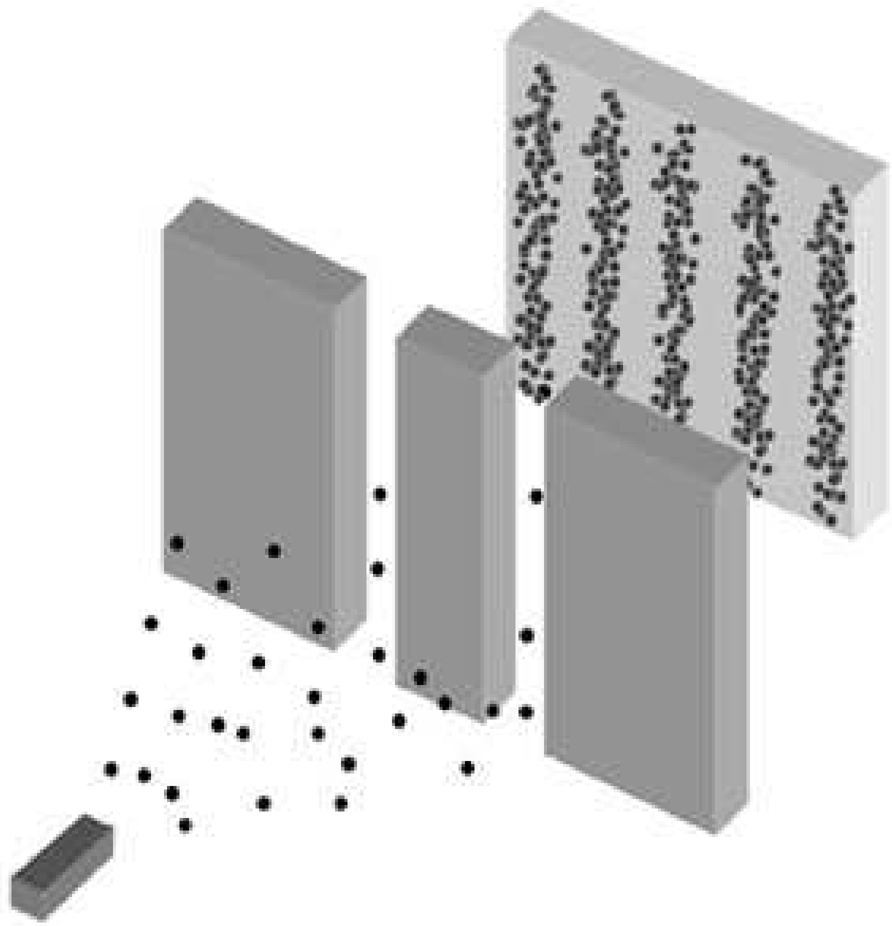
Double-Slit Experiment (Young's Experiment)

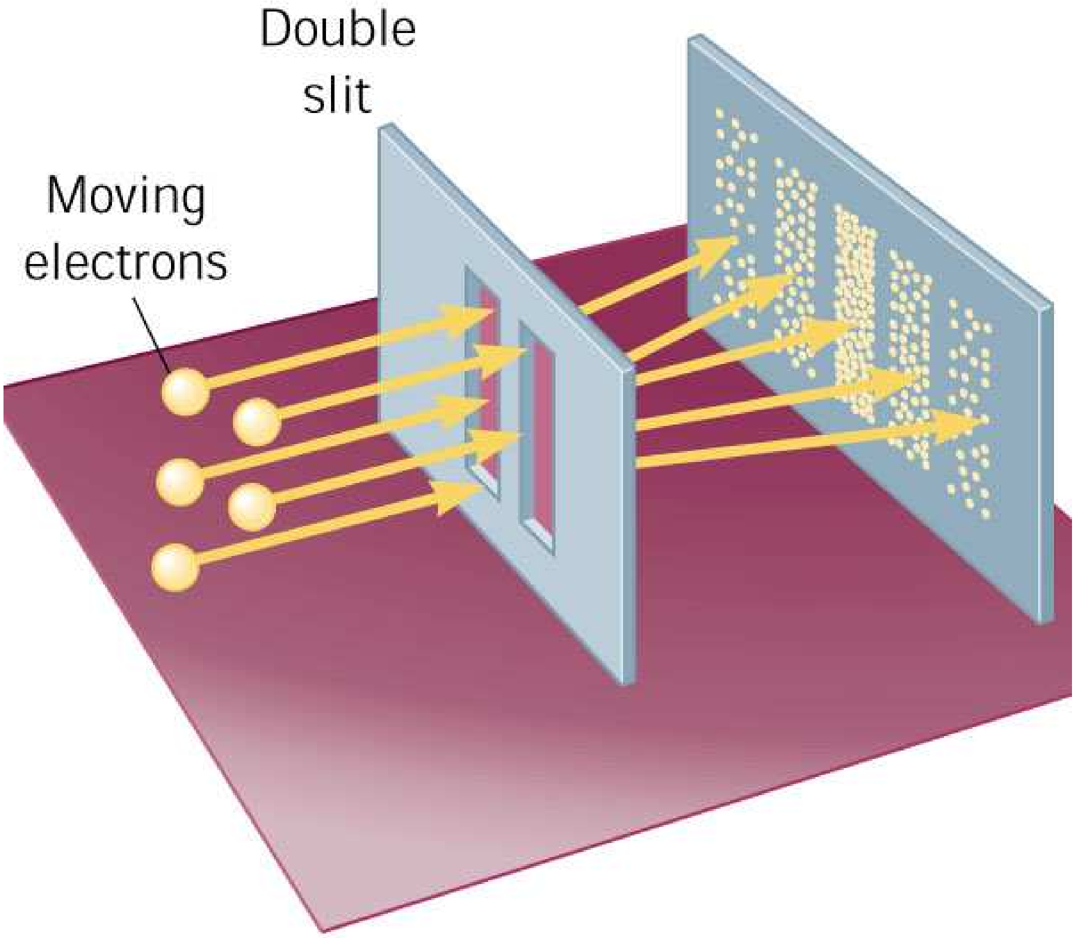


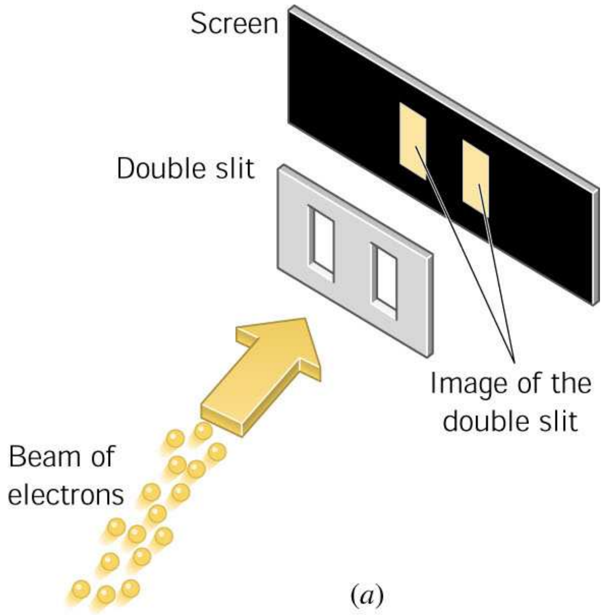
Electrons Through Double-Slit (Classical Physics)

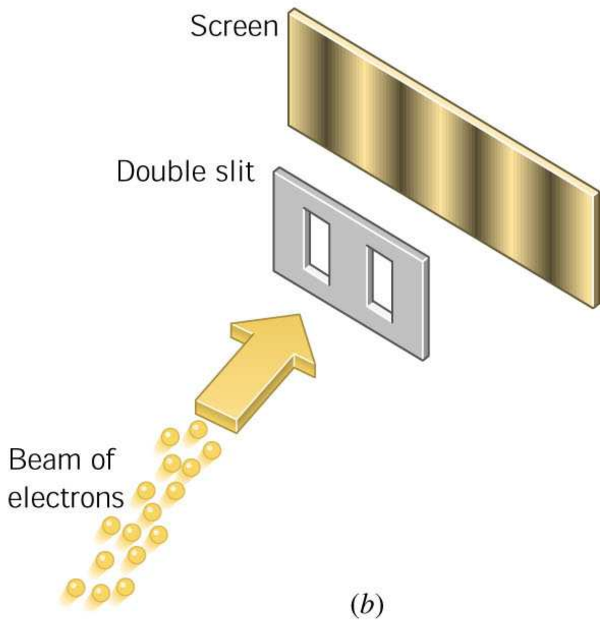


Electrons Through Double-Slit (Quantum Physics)




Double
slit
Moving
electrons


Screen
Double slit
Image of the
double slit
Beam of
electrons
(*a*)


Screen
Double slit
Beam of
electrons
(*b*)

Superposition Principle

- **An example:**wave function of a moving electron is represented by a packet of waves.

- The states of a micro-particle are combined in groups.

- Importance of interfering amplitudes $\Longrightarrow$ concept of superposition of states.

- Any state from one set may be represented in the form of a superposition of states from another set.

- $\psi_\alpha = \sum_\beta \phi_{\alpha\beta}\psi_\beta$.

- $\phi_{\alpha\beta}$is the probability amplitude that a particle in state ψ_α may also be found in state ψ_β.

- $\phi_{\alpha\beta} = 0 \quad \Longrightarrow$ Orthogonal states.

- Superposition principle supplements uncertainty relation.

- Particle state:ψ_α- some physical quantities are measurable; then the values of them in ψ_β may be predicted with probability $\mid \phi_{\alpha\beta} \mid^2$.

- Mathematical point of view: Classical superposition and quantum superposition are more or less analogous.

- Classical case:$\phi_{\alpha\beta}$ are always zero, which is not true for quantum case. Classical concept of superposition of states is equivalent to a destruction of quantum mechanical superposition

- Classical examples: Vibrating strings, membranes, etc. $\longrightarrow$ Fourier analysis.

Superposition Principle and Polarization Experiment

- Photon states: $\psi = \cos\theta\, \psi_1 + \sin\theta\, \psi_2$.
- $\sin\theta$-term $\perp$ optic axis $\Longrightarrow$ can not pass; $\cos\theta$-term $||$-can emerge.
- Single photon- one can not predict.

Superposition Principle and Double Slit Experiment

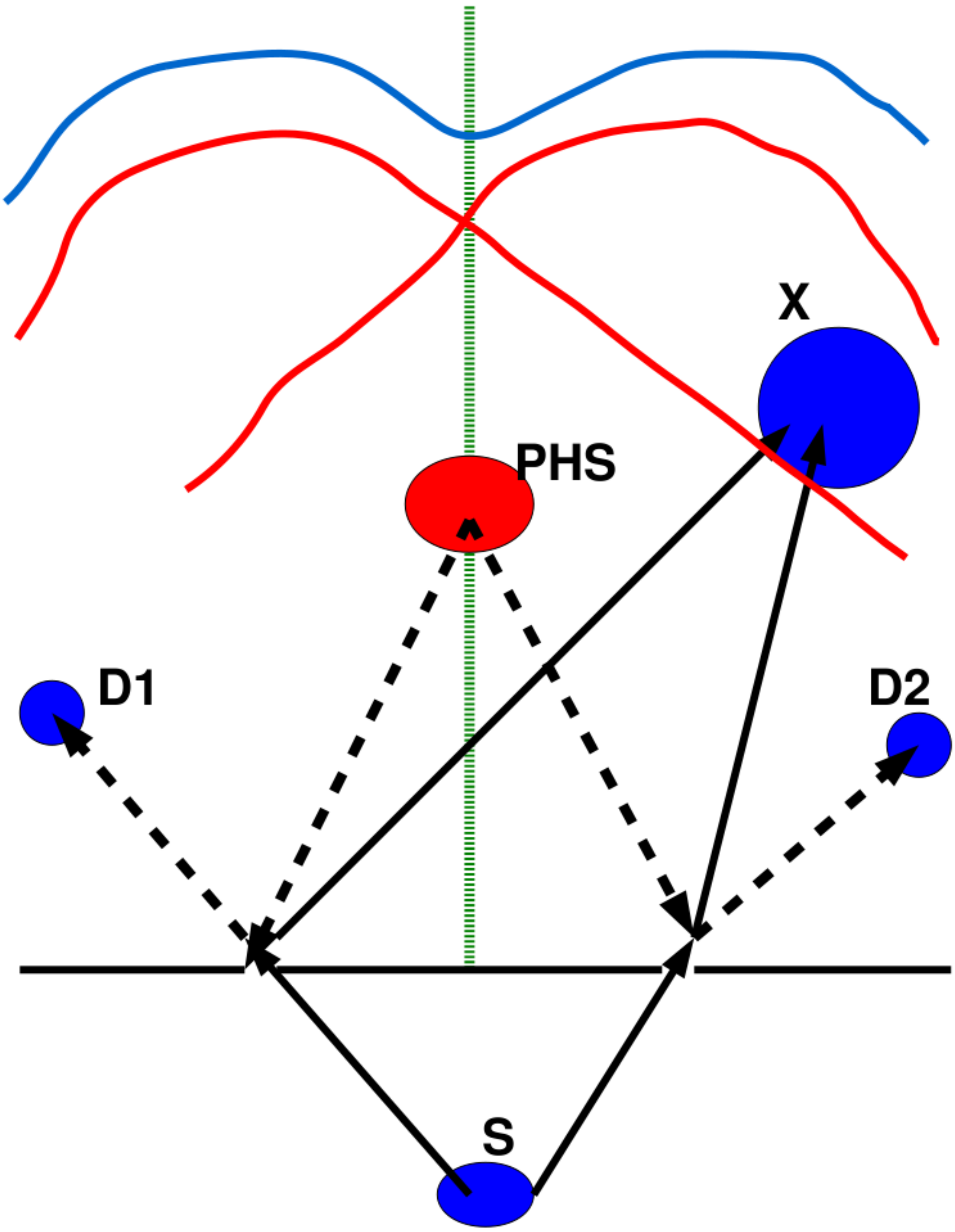
- Wave functions:$\psi_s = C_A\psi_A + C_B\psi_B \longrightarrow$ Initial State.

- Wave functions:$\psi_f = C_A\psi_A + C_B\psi_B \longrightarrow$ Final State.

- Transition amplitude:ϕ_{sf} gives interference pattern.

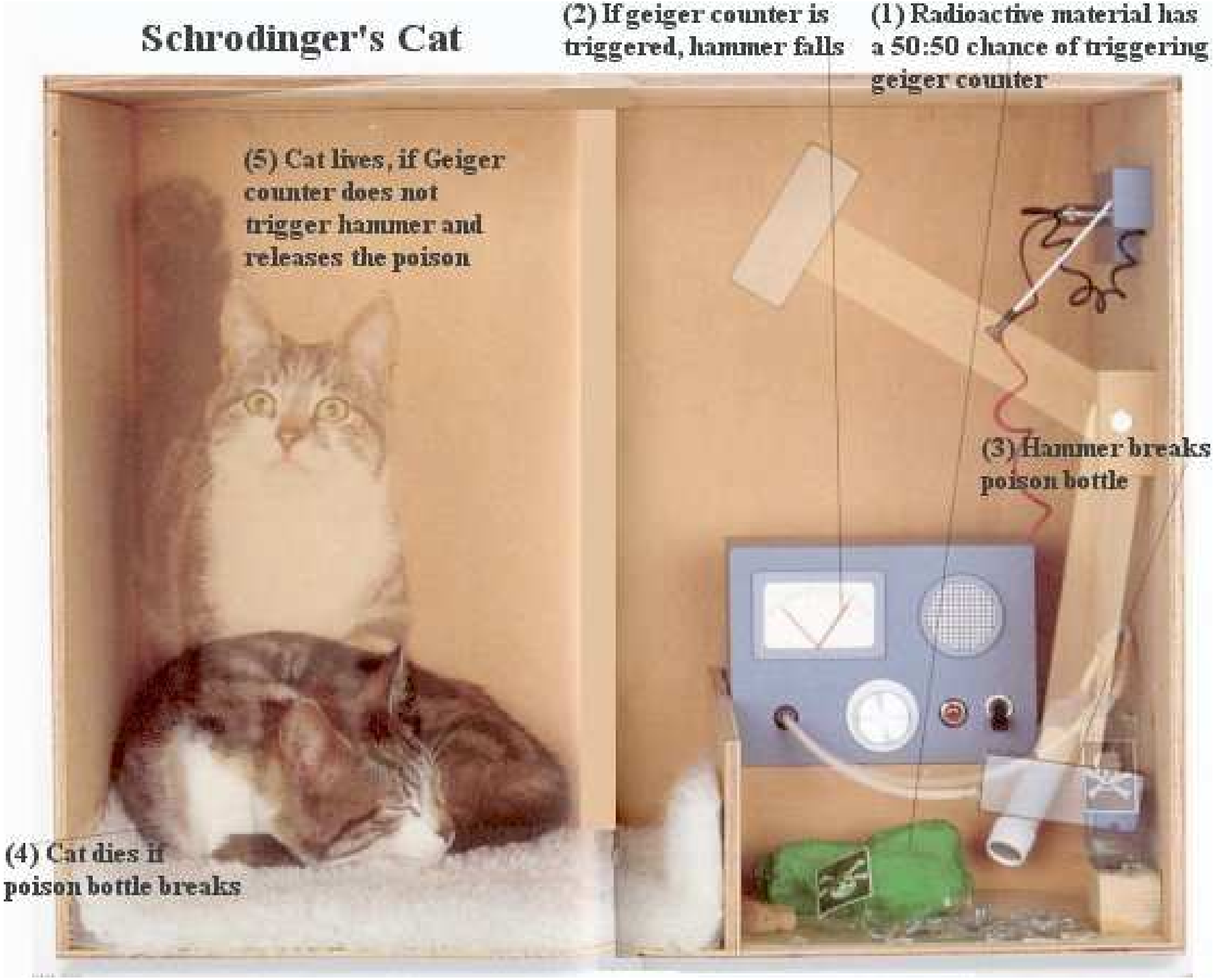
- $\mid \phi_{sf} \mid^2 \propto \mid C_A\psi_A \mid^2 + \mid C_B\psi_B \mid^2 \longrightarrow$ when you try to see electrons at the slits $\Longrightarrow$ Completely distinguishable.

- $\mid \phi_{sf} \mid^2 \propto \mid C_A\psi_A + C_B\psi_B \mid^2 \longrightarrow$ when you don't try to see electrons at the slits $\Longrightarrow$ Completely indistinguishable.


X
PHS
D1
D2
S

QM is Probabilistic in Nature

Famous Schrödinger's Cat Thought Experiment (1935)



Thought Experiment

A cat is placed in a box, together with an alpha-active radioactive atom of life time say t sec. If the atom decays, and the Geiger-counter detects an alpha particle, the hammer hits a flask of Hydro-Cyanic acid (HCN), killing the cat. The paradox lies in the clever coupling of quantum and classical domains. Before the observer opens the box at time t, the cat's fate is tied to the wave function of the atom, which is itself in a superposition of decayed and undecayed states. Thus, said Schrödinger, the cat must itself be in a superposition of dead and alive states before the observer opens the box.

The Biography of Quantum Mechanics- in Brief

- During the last about hundred years, quantum mechanics has passed primarily through three stages of development.

- The first stage: Planck to de Broglie- about 25 years $\longrightarrow$ material nature of light waves to wave nature of matter.

- The Second stage: Discovery of matter wave to second World War-II; Development of QM based on de Broglie's discovery; Schrödinger gave his famous equation; Heisenber developed it through an alternative way and gave uncertainty relation; Dirac synthesized QM and STR of Einstein; Atomic phenomena are almost explained; Theory of the atomic nucleus was created $\longrightarrow$ birth of Nuclear Physics.

- The Third stage: After World War-II; QM was extended to the elementary particles and fields $\longrightarrow$ birth of Particle Physics and Quantum Field Theory.

- In the third stage: After the brilliant victory of its early years came a series of setbacks and failures.

- The impression is that, good as it is against atoms and molecules, it is simply not strong enough for nuclear physics, elementary particles and their structures and interactions.

- The strong interaction problem could not be solved.

- Could not reproduce the ground state properties of strongly interacting particles.

Some Specialized Remarks

- It may need an extension of the framework, perhaps new important non-contradictory propositions to be added to make the theory stronger.

- We know that there have never been all powerful theories and there never will be.

- Like each theory, Quantum Physics had a shaky childhood, strong youth, when it resolves a dozones of extrahard problems.

- Today quantum mechanics has reached its peak of maturity and old age is creeping up.

- Due to calm maturity, the forward movement of the theory has slowed down. The theory has spreaded over broader spheres of phenomena, moving into technology and industry and establishing contacts with other disciplines.

- **Finally, today quantum mechanics is the strongest theory of physical science. It has solved a lot of physical problems ranging from the stellar interior structure, stellar evolution to that of atoms, molecules, atomic nuclei and elementary particle.**